United Kingdom
Debt
Management
Office

Eastcheap Court 11 Philpot Lane London EC3M 8UD

# UNITED KINGDOM DEBT MANAGEMENT OFFICE

# Formulae for Calculating Gilt Prices from Yields

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## UK DEBT MANAGEMENT OFFICE FORMULAE FOR CALCULATING GILT PRICES FROM YIELDS

This paper sets out the United Kingdom Debt Management Office's (DMO) formulae for calculating gilt prices from gross redemption yields, thus allowing a formal settlement convention to be applied to a trade conducted on a yield basis. The formulae in this paper are effective from 1 November 1998. This is an updated version of the paper first published in June 1998. In this version, minor changes have been made although the formulae themselves are largely unchanged (except for the removal of partly-paid terms). In particular, the section on accrued interest (Section Two) has been expanded to include formulae for accrued interest during short and long first coupon periods for conventional and indexlinked gilts.

In the event that the formulae are to be used to derive yields from prices it is not possible (in most cases) to solve for yield in terms of price algebraically, and so some form of numerical technique<sup>1</sup> must be used if, given a price, a value for the redemption yield is required.

The first section of the paper states the formulae that will be used; these are split into the different classes of gilt (new formulae for new instruments will be added to the paper as and when required). For the purpose of this paper, "cash flows" refer to cash flows receivable by the buyer of the gilt. Also, "nearest rounding" to, say, six decimal places means round the sixth decimal place up by one if the seventh decimal place is 5 or above, and then truncate at the sixth decimal place.

Compounding will occur on quasi-coupon dates. Quasi-coupon dates are the dates on the semi-annual cycle (or quarterly cycle, for quarterly paying gilts) defined by the maturity date, irrespective of whether cash flows occur on those dates (examples of quasi-coupon dates on which cash flows would not occur include the first quasi-coupon date of a new issue having a long first dividend period; the next quasi-coupon date of a gilt settling in its ex-dividend period; and most quasi-coupon dates of a strip). The quasi-coupon dates for undated gilts are defined by their regular coupon cycle. A full (quasi-) coupon period is defined as the period between two consecutive quasi-coupon dates. For example, a gilt settling on its issue date (assuming this is not also a quasi-coupon date) will have a quasi-coupon date following

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the issue date. Cash flows and quasi-coupon dates which are due to occur on non-business days are not

adjusted.

This means that cash flows which occur off quasi-coupon dates (such as some early redemption

payments on double-dated or undated gilts) will have an additional fractional period associated with

their discounting process to allow for discounting back (i.e. towards the settlement date) by a fractional

period to the quasi-coupon date immediately prior to their occurrence, before being discounted back to

the settlement date.

All settlement values derived from these formulae (yield to price) should be rounded to the nearest

penny on the trade. In addition, the price/yield formulae discount all cash flows on the quasi-coupon

cycle using the 'actual' daycount convention: this is consistent with the agreed market consensus

for discounting the cash flow from a strip.

Following market consultation, the inflation assumption that will be used in the formulae for index-

linked gilts is 3% per annum. This will be reviewed by the DMO as and when a majority of market

participants judge that a review is necessary.

The second section provides formulae for the calculation of accrued interest. An annex describing the

procedure for estimating the nominal values of unknown future cash flows on index-linked gilts and on

Floating Rate Gilts can be found at the end of the paper.

Any questions on this paper should be addressed to:

Gurminder Bhachu (gurminder.bhachu@dmo.gsi.gov.uk)

UNITED KINGDOM DEBT MANAGEMENT OFFICE

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#### SECTION ONE: PRICE/YIELD FORMULAE

Conventional Gilts; Double-dated and Undated Gilts with Assumed (or Actual) Redemption on a Quasi-Coupon Date<sup>2</sup>

The formula for calculating the price from the yield is given by:

$$P = v^{\frac{r}{s}} \left( d_1 + d_2 v + \frac{cv^2}{f(1-v)} (1-v^{n-1}) + 100v^n \right) \quad \text{for } n \ge 1$$

Where:  $P = \text{Dirty price per } £100 \text{ nominal of the gilt.}^3$ 

= Cash flow due on next quasi-coupon date, per £100 nominal of the gilt (may be zero if the gilt has a long first dividend period or if the gilt settles in its exdividend period; or may be greater or less than  $\frac{c}{2}$  during long or short first dividend periods respectively).

 $d_2$  = Cash flow due on next but one quasi-coupon date, per £100 nominal of the gilt (may be greater than  $\frac{c}{2}$  during long first dividend periods).

c = Coupon per £100 nominal of the gilt.

y = Nominal redemption yield (decimal) i.e. if the yield is 5% then y = 0.05.

f = Number of coupons payable on the gilt per year (f will be equal to 2 or 4).

 $v = \frac{1}{1 + \frac{y}{f}}$ 

r = Number of calendar days from the settlement date to the next quasi-coupon date.

s = Number of calendar days in the full coupon period in which the settlement date
 occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date).

n =Number of full coupon periods between the next quasi-coupon date and

redemption.

For n = 0, the equation reduces to

$$P = v^{\frac{r}{s}} (d_1 + 100)$$

#### **Index-linked Gilts**

(1) Not all the nominal values of future cash flows are fixed

Case 1: Two or more cash flows remaining

The formula for calculating the price from the yield is given by:

$$P = \left[ d_1 + d_2(uw) + \frac{acw^2}{2(1-w)} (1-w^{n-1}) \right] (uw)^{\frac{r}{s}} + 100au^{\frac{r}{s}} w^{\frac{r}{s}}$$
 for  $n \ge 1$ 

Where:  $P = \text{Dirty price per £100 nominal of the gilt}^3$ .

= Cash flow due on next quasi-coupon date, per £100 nominal of the gilt (may be zero if the gilt has a long first dividend period or if the gilt settles in its exdividend period; or may be greater or less than  $\frac{c}{2}$  during long or short first dividend periods respectively).

 $d_2$  = Cash flow due on next but one quasi-coupon date, per £100 nominal of the gilt (may be greater than  $\frac{c}{2}$  times the RPI Ratio during long first dividend periods)<sup>4</sup>.

c = (Real) coupon per £100 nominal.

r = Number of calendar days from the settlement date to the next quasi-coupon

date.

s = Number of calendar days in the full coupon period in which the settlement date
 occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date).

 $\rho$  = Semi-annually compounded real redemption yield (decimal) i.e. if the real yield is 3% then  $\rho = 0.03$ .

$$w = \frac{1}{1 + \frac{\rho}{2}}$$

 $\pi$  = The assumed annual inflation rate (decimal) = 0.03.

$$u = \left(\frac{1}{1+\pi}\right)^{\frac{1}{2}} = \left(\frac{1}{1.03}\right)^{\frac{1}{2}}$$

n = Number of full coupon periods between the next quasi-coupon date and redemption.

RPIB = The base RPI for the stock i.e. the RPI scheduled to be published seven months prior to the month of issue of the gilt and relating to the month before that prior month (for example, if the stock is issued in November then its base RPI is the RPI for March of that year).

*RPIL* = The latest published RPI at the time of settlement.

k = Number of months between the month of the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the month of the latest published RPI at the time of settlement. For example, if the RPI for January is the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the latest published RPI at the time of settlement is the RPI for April, then k = 3.

$$a = \frac{RPIL}{RPIB} \cdot u^{\frac{2k}{12}}$$

Case 2: One cash flow remaining (i.e. the final dividend and redemption payment)

If the RPI determining the redemption value is published <u>after</u> the stock goes ex-dividend for the penultimate time, the price/yield formula is defined as:

$$P = \left(100 + \frac{c}{2}\right) \cdot \frac{a}{u} \cdot \left(uw\right)^{\frac{r}{s} + \alpha}$$

Where:  $P = \text{Dirty price per £100 nominal of the gilt}^3$ .

c = (Real) coupon per £100 nominal.

 $\rho$  = Real redemption yield (decimal) i.e. if the yield is 3% then  $\rho$  =0.03.

$$w = \frac{1}{1 + \frac{\rho}{2}}$$

 $\pi$  = The assumed annual inflation rate (decimal) = 0.03.

$$u = \left(\frac{1}{1+\pi}\right)^{\frac{1}{2}} = \left(\frac{1}{1.03}\right)^{\frac{1}{2}}$$

Number of calendar days from the settlement date to the next quasi-coupon date.

s = Number of calendar days in the full coupon period in which the settlement date occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date).

 $\alpha = \begin{cases} 1 & \text{if the gilt is settling in its penultimate ex - dividend period} \\ 0 & \text{if the gilt is settling after its penultimate quasi-coupon date} \end{cases}$ 

RPIB = The base RPI for the stock i.e. the RPI scheduled to be published seven months prior to the month of issue of the gilt and relating to the month before that prior month (for example, if the stock is issued in November then its base RPI is the RPI for March of that year).

*RPIL* = The latest published RPI at the time of settlement.

Rumber of months between the month of the RPI that defines the dividend due
 (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the month of the latest published RPI at the time of settlement. For example, if the RPI for
 January is the RPI that defines the dividend due (or would ordinarily be due, in

the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the latest published RPI at the time of settlement is the RPI for April, then k=3.

$$a = \frac{RPIL}{RPIB} \cdot u^{\frac{2k}{12}}$$

In this special case, we can solve algebraically for yield in terms of price:

$$\rho = 2 \cdot \left( u \cdot \left( \frac{uP}{\left( 100 + \frac{c}{2} \right) \cdot a} \right)^{-\frac{s}{r + \alpha s}} - 1 \right)$$

#### (2) Nominal values of <u>all</u> future cash flows are fixed

Case 1: Index-linked stocks that have passed both their penultimate ex-dividend date <u>and</u> the date on which the RPI determining the final redemption payment is published provide a known cash flow on just one remaining date. The price/yield formula in this case is:

$$P = v^{\frac{r}{s} + \alpha} \left( d_{LAST} + R \right)$$

Where: P =

P = Dirty price per £100 nominal of the gilt<sup>3</sup>.

 $d_{\it LAST}$  = Final dividend payment per £100 nominal of the gilt.

R = Final redemption payment per £100 nominal of the gilt.

y = Semi-annually compounded <u>nominal</u> redemption yield (decimal) i.e. if the yield is 5% then y=0.05.

$$v = \frac{1}{1 + \frac{y}{2}}$$

r = Number of calendar days from the settlement date to the next quasi-coupon date.

s = Number of calendar days in the full coupon period in which the settlement date occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date).

 $\alpha = \begin{cases} 1 & \text{if the gilt is settling in its penultimate ex - dividend period} \\ 0 & \text{if the gilt is settling after its penultimate quasi-coupon date} \end{cases}$ 

In this special case, we can solve algebraically for yield in terms of price:

$$y = 2 \cdot \left[ \left( \frac{P}{d_{LAST} + R} \right)^{-\frac{s}{r + \alpha s}} - 1 \right]$$

Case 2: When valuing index-linked stocks on days between the publication date of the RPI determining the redemption payment and the penultimate ex-dividend date (assuming that the RPI determining the redemption value is published <u>before</u> the stock goes ex-dividend for the penultimate time), the price/yield formula is defined as:

$$P = (d_{PEN} + (d_{LAST} + R) \cdot v)v^{\frac{r}{s}}$$

Where:

P = Dirty price per £100 nominal of the gilt<sup>3</sup>.

 $d_{PEN}$  = Penultimate dividend payment per £100 nominal of the gilt.

 $d_{LAST}$  = Final dividend payment per £100 nominal of the gilt.

y = Semi-annually compounded <u>nominal</u> redemption yield (decimal) i.e. if the yield is 5% then y = 0.05.

 $v = \frac{1}{1 + \frac{y}{2}}$ 

R = Redemption payment per £100 nominal of the gilt.

r = Number of calendar days from the settlement date to the next quasi-coupon date.

s = Number of calendar days in the full coupon period in which the settlement date occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date).

#### Double-dated Gilts

A double-dated gilt has a final maturity date and in addition an earlier maturity date, with Her Majesty's Treasury having the right to redeem the gilt on any day between these two dates, provided that the relevant notice is given (usually 3 months). In order to calculate the redemption yield for such gilts it is necessary to make some assumption about when the gilt will be redeemed (where a specific redemption date has not yet been announced by the authorities). The convention used in these formulae is referred to as the *yield/coupon rule*:

Case 1: The settlement date is more than x months before the first date in the redeemable band (where x is the period of notice required to be given to call the gilt as specified in its prospectus - usually 3 months). Then if the nominal redemption yield y is greater than or equal to the coupon, the latest redemption date in the redeemable band is assumed; otherwise the earliest redemption date in the redeemable band is assumed.

Case 2: The settlement date is either less than x months before the first date in the redeemable band (where x is the period of notice required to be given to call the gilt as specified in its prospectus - usually 3 months), or the settlement date is in the redeemable band. Then if notice has not yet been given by the authorities that the gilt will be redeemed early, the latest redemption date in the redeemable band is assumed (irrespective of whether the nominal redemption yield y is greater than or less than the coupon).

Having made such an assumption about the redemption date, if this is on a quasi-coupon date the formula for conventional gilts should be used; if this is on a date which is not a quasi-coupon date, the following formula should be used:

$$P = v^{\frac{r}{s}} \left( d_1 + d_2 v + \frac{cv^2}{2(1-v)} (1-v^{n-1}) + \left(100 + d_f\right) \cdot v^n \cdot v^{\frac{t}{u}} \right)$$
 for  $n \ge 1$ 

Where:  $P = \text{Dirty price per } £100 \text{ nominal of the gilt}^3$ .

 $d_1$  = Cash flow due on next quasi-coupon date, per £100 nominal of the gilt (may be

zero if the gilt has a long first dividend period or if the gilt settles in its exdividend period; or may be greater or less than  $\frac{c}{2}$  during long or short first dividend periods respectively).

- $d_2$  = Cash flow due on next but one quasi-coupon date, per £100 nominal of the gilt (may be greater than  $\frac{c}{2}$  during long first dividend periods).
- $d_f$  = Partial coupon due on off quasi-coupon redemption date, per £100 nominal of the gilt.
- c = Coupon per £100 nominal of the gilt.
- y = Semi-annually compounded nominal redemption yield (decimal) i.e. if the yield is 5% then y=0.05.
- $v = \frac{1}{1 + \frac{y}{2}}$
- r = Number of calendar days from the settlement date to the next quasi-coupon date.
- s = Number of calendar days in the full coupon period in which the settlement date
   occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date).
- t = Number of calendar days from the redemption date to the preceding quasicoupon date.
- = Number of calendar days in the full coupon period in which the redemption dateoccurs.
- n = Number of full coupon periods between the next quasi-coupon date and redemption.

For n = 0, the equation reduces to:

(1) If the period between the settlement date and the redemption date spans a quasi-coupon date

$$P = v^{\frac{r}{s}} \left( d_1 + \left( 100 + d_f \right) \cdot v^{\frac{t}{u}} \right)$$

(2) If the period between the settlement date and the redemption date does not span a quasi-coupon date

$$P = \left(100 + d_f\right) \cdot v^{\frac{t^*}{u}}$$

Where: t = Number of calendar days from the settlement date to the redemption date.

#### **Undated Gilts**

All current undated gilts in issue have a date after which they can be redeemed (for example, 3 1/2% War Loan is dated '1952 or after'). In order to calculate the redemption yield for such gilts it is necessary to make some assumption about when the gilt will be redeemed (where a specific redemption date has not yet been announced by the authorities).

If notice has not yet been given by the authorities that the gilt will be redeemed early, it is assumed that the gilt will not be redeemed and the infinite cash flow formula should be used (see below), irrespective of whether the nominal redemption yield *y* is greater than or less than the coupon.

For an assumed or actual early redemption date, if this is on a quasi-coupon date the formula for conventional gilts should be used; if this is on a date which is not a quasi-coupon date, the following formula should be used:

$$P = v^{\frac{r}{s}} \left( d_1 + d_2 v + \frac{cv^2}{f(1-v)} \left( 1 - v^{n-1} \right) + \left( 100 + d_f \right) \cdot v^n \cdot v^{\frac{t}{u}} \right)$$
 for  $n \ge 1$ 

Where:  $P = \text{Dirty price per } £100 \text{ nominal of the gilt}^3$ .

- $d_1$  = Cash flow due on next quasi-coupon date, per £100 nominal of the gilt (may be zero if the gilt has a long first dividend period or if the gilt settles in its exdividend period; or may be greater or less than  $\frac{c}{2}$  during long or short first dividend periods respectively).
- $d_2$  = Cash flow due on next but one quasi-coupon date, per £100 nominal of the gilt (may be greater than  $\frac{c}{2}$  during long first dividend periods).
- $d_f$  = Partial coupon due on off quasi-coupon redemption date, per £100 nominal of the gilt.
- c = Coupon per £100 nominal of the gilt.
- y = Nominal redemption yield (decimal) i.e. if the yield is 5% then y = 0.05.
- f = Number of coupons payable on the gilt per year (f will be equal to 2 or 4).
- $v = \frac{1}{1 + \frac{y}{f}}$
- r = Number of calendar days from the settlement date to the next quasi-coupon date.
- s = Number of calendar days in the full coupon period in which the settlement date occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date).
- t = Number of calendar days from the redemption date to the preceding quasicoupon date.
- = Number of calendar days in the full coupon period in which the redemption date occurs.
- n = Number of full coupon periods between the next quasi-coupon date and redemption.

For n = 0, the equation reduces to:

(1) If the period between the settlement date and the redemption date spans a quasi-coupon date

$$P = v^{\frac{r}{s}} \left( d_1 + \left( 100 + d_f \right) \cdot v^{\frac{t}{u}} \right)$$

(2) If the period between the settlement date and the redemption date does not span a quasi-coupon date

$$P = (100 + d_f) \cdot v^{\frac{t^*}{u}}$$

Where: t = Number of calendar days from the settlement date to the redemption date.

Infinite cash flow method: For an infinite set of cash flows (i.e. where it is assumed that the gilt will not be redeemed early) we use the formula for a conventional gilt and take P to be the limit of the sum of the discounted cash flows as n (the number of full coupon periods between the next quasi-coupon date and redemption) tends to infinity. Since |v| < 1, this limit exists and is equal to

$$P = v^{\frac{r}{s}} \left( d_1 + d_2 v + \frac{cv^2}{f(1-v)} \right)$$

Where:  $P = \text{Dirty price per £100 nominal of the gilt}^3$ .

 $d_1$  = Cash flow due on next quasi-coupon date, per £100 nominal of the gilt (may be zero if the gilt has a long first dividend period or if the gilt settles in its exdividend period; or may be greater or less than  $\frac{c}{2}$  during long or short first dividend periods respectively).

 $d_2$  = Cash flow due on next but one quasi-coupon date, per £100 nominal of the gilt

(may be greater than  $\frac{c}{2}$  during long first dividend periods).

c = Coupon per £100 nominal of the gilt.

y = Nominal redemption yield (decimal) i.e. if the yield is 5% then y = 0.05.

f = Number of coupons payable on the gilt per year (f will be equal to 2 or 4).

$$v = \frac{1}{1 + \frac{y}{f}}$$

r = Number of calendar days from the settlement date to the next quasi-coupon date.

S = Number of calendar days in the full coupon period in which the settlement date
 occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date).

#### Floating Rate Gilts

Unlike conventional gilts, the coupons on Floating Rate Gilts are not fixed but vary in line with a money market index (hereafter referred to as Index)<sup>5</sup>. Instead of computing a redemption yield for such gilts we calculate the discount at which they trade relative to Index. Given a discount margin y the price is computed using the formula below. It is not possible to solve for the discount margin in terms of price algebraically, and so some form of numerical technique<sup>1</sup> must be used if, given a price, a value for the discount margin relative to Index is required.

$$P = \frac{d_1}{\left(1 + \frac{(m+y)q}{36500}\right)} + \frac{1}{\left(1 + \frac{(m+y)q}{36500}\right)} \left[ \sum_{k=1}^{n} \frac{\frac{r_k \times (L+D)}{365}}{\prod_{j=1}^{k} \left(1 + \frac{r_j \times (L+y)}{36500}\right)} + \frac{100}{\prod_{j=1}^{n} \left(1 + \frac{r_j \times (L+y)}{36500}\right)} \right]$$

Where:  $P = \text{Dirty price per £100 nominal of the gilt}^3$ .

 $d_1$  = Cash flow due on next quasi-coupon date, per £100 nominal of the gilt<sup>6</sup>.

q = Number of calendar days from the settlement date to the next quasi-coupon date.  $r_k$  = Number of calendar days in the kth coupon period, where the coupon period after the coupon period in which the settlement date occurs is defined as the first coupon period (k is between 1 and n).

n = Number of full coupon periods between the next quasi-coupon date and redemption.

The latest value of *Index* (rounded to the nearest 5th decimal place) that is
 used as the reference to set future coupon payments, quoted as a percentage.

m = The latest q-day Index rate (rounded to the nearest 5th decimal place), quoted as a percentage. For the purpose of this paper, set m = L.

= The discount (negative) or premium (positive) at which the coupon is fixed relative to *Index* (e.g. for a Floating Rate Gilt of coupon *Index*  $-\frac{1}{8}$ ,  $D = -\frac{1}{8}$ ).

y = The discount (negative) or premium (positive) at which the gilt trades relative to *Index*, quoted as a percentage.

#### **Strips**

In February 1997 the Bank of England (as UK government debt manager at that time) published a consultative paper seeking views on what standardised formula for computing market prices from gross redemption yields should be adopted to allow gilt strips to trade on a yield basis. The result of the consultation was indicated by Press Notices on 30 May 1997 and 12 June 1997. The market consensus was that the following method was the most suitable for strips:

$$P = \frac{100}{\left(1 + \frac{y}{2}\right)^{\frac{r}{s} + n}}$$

Where: P = Price per £100 nominal of the strip.

y =Strip gross redemption yield (decimal) i.e. if the yield is 5% then y = 0.05.

r = Number of calendar days from the settlement date to the next quasi-coupon date.

s = Number of calendar days in the quasi-coupon period in which the settlement date

occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date).

n = Number of full coupon periods between the next quasi-coupon date and redemption.

The settlement proceeds are rounded to the nearest penny on the traded nominal amount (with no intermediate rounding of price).

In this special case, we can solve algebraically for yield in terms of price:

$$y = 2 \cdot \left[ \left( \frac{100}{P} \right)^{\frac{s}{r+ns}} - 1 \right]$$

#### SECTION TWO: CALCULATION OF ACCRUED INTEREST

While coupon payments on individual gilts are usually made only twice a year, gilts can be traded on any business day. Whenever a gilt changes hands on a day that is not a coupon payment date, the valuation of the gilt will reflect the proximity of the next coupon payment date. This is effected by the payment of <u>accrued interest</u> to compensate the seller for the period since the last coupon payment date during which the seller has held the gilt but for which he receives no coupon payment. The accrued interest for gilts is computed as follows<sup>8</sup> (based on the 'actual/actual' daycount convention effective from 1 November 1998):

#### (1) Standard dividend periods

(i) All gilts excluding Floating Rate Gilts which do not have an ex-dividend period

 $AI = \begin{cases} \frac{t}{s} \cdot d_1 & \text{if the settlement date occurs on or before the ex - dividend date} \\ \left(\frac{t}{s} - 1\right) \cdot d_1 & \text{if the settlement date occurs after the ex - dividend date} \end{cases}$ 

Where: AI =Accrued interest per £100 nominal of the gilt.

 $d_1$  = Next dividend per £100 nominal of the gilt, as published.

t = Number of calendar days from the last dividend date to the settlement date.

s = Number of calendar days in the full coupon period in which the settlement date occurs.

(ii) Floating Rate Gilts which do not have an ex-dividend period

$$AI = \frac{t}{s} \cdot d_1$$

Where: AI =Accrued interest per £100 nominal of the gilt.

 $d_1$  = Next dividend per £100 nominal of the gilt, as published.

t = Number of calendar days from the last dividend date to the settlement date.

s = Number of calendar days in the full coupon period in which the settlement date
 occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date).

#### (2) Short first dividend periods

#### (i) Conventional gilts

 $AI = \begin{cases} \frac{t^*}{s} \times \frac{c}{2} & \text{if the settlement date occurs on or before the ex - dividend date} \\ \left(\frac{t^* - r}{s}\right) \times \frac{c}{2} & \text{if the settlement date occurs after the ex - dividend date} \end{cases}$ 

Where: AI =Accrued interest per £100 nominal of the gilt.

c = Coupon per £100 nominal of the gilt.

 $t^*$  = Number of calendar days from the issue date to the settlement date.

s = Number of calendar days in the full coupon period in which the settlement date occurs.

r = Number of calendar days from the issue date to the next (short) coupon date.

#### (ii) Index-linked gilts

 $AI = \begin{cases} \frac{t^*}{s} \times \frac{c}{2} \times \frac{RPID}{RPIB} & \text{if the settlement date occurs on or before the ex - dividend date} \\ \left(\frac{t^*-r}{s}\right) \times \frac{c}{2} \times \frac{RPID}{RPIB} & \text{if the settlement date occurs after the ex - dividend date} \end{cases}$ 

Where: AI =Accrued interest per £100 nominal of the gilt.

c = Coupon per £100 nominal of the gilt.

 $t^*$  = Number of calendar days from the issue date to the settlement date.

s = Number of calendar days in the full coupon period in which the settlement date occurs.

r = Number of calendar days from the <u>issue</u> date to the next (short) coupon date.

RPID = The RPI which fixes the first dividend payment for the gilt i.e. the RPI scheduled to be published seven months prior to the month of the first dividend payment and relating to the month before that prior month (for example, if the first dividend payment on the gilt will be in November then the RPI which fixes its value is the RPI for March of that year).

RPIB = The base RPI for the gilt i.e. the RPI scheduled to be published seven months prior to the month of issue of the gilt and relating to the month before that prior month (for example, if the gilt is issued in November then its base RPI is the RPI for March of that year).

#### (3) Long first dividend periods

#### (i) Conventional gilts

 $AI = \begin{cases} \frac{t}{s_1} \times \frac{c}{2} & \text{if the settlement date occurs during the first coupon period} \\ \frac{r_1}{s_1} + \frac{r_2}{s_2} \times \frac{c}{2} & \text{if the settlement date occurs during the second coupon period on or before the ex-dividend date} \\ \frac{r_2}{s_2} - 1 \times \frac{c}{2} & \text{if the settlement date occurs during the second coupon period after the ex-dividend date} \end{cases}$ 

Where: AI =Accrued interest per £100 nominal of the gilt.

c = Coupon per £100 nominal of the gilt.

= Number of calendar days from the <u>issue</u> date to the settlement date in the first coupon period (this term only applies if the gilt settles in the first coupon period).

 $s_1$  = Number of calendar days in the full coupon period in which the <u>issue</u> date occurs.

s<sub>2</sub> = Number of calendar days in the full coupon period after the coupon period in
 which the <u>issue</u> date occurs.

 $r_1$  = Number of calendar days from the <u>issue</u> date to the next (theoretical) coupon date.

= Number of calendar days from the (theoretical) coupon date after the <u>issue</u> date to the settlement date in the coupon period after the coupon period in which the <u>issue</u> date occurs (this term only applies if the gilt settles in the second coupon period.

#### (ii) Index-linked gilts

$$AI = \begin{cases} \frac{t}{s_1} \times \frac{c}{2} \times \frac{RPID}{RPIB} & \text{if the settlement date occurs during the first coupon period} \\ \frac{r_1}{s_1} + \frac{r_2}{s_2} \times \frac{c}{2} \times \frac{RPID}{RPIB} & \text{if the settlement date occurs during the second coupon period on or before the ex-dividend date} \\ \frac{r_2}{s_2} - 1 \times \frac{c}{2} \times \frac{RPID}{RPIB} & \text{if the settlement date occurs during the second coupon period after the ex-dividend date} \end{cases}$$

Where: AI =Accrued interest per £100 nominal of the gilt.

c = Coupon per £100 nominal of the gilt.

= Number of calendar days from the <u>issue</u> date to the settlement date in the first coupon period (this term only applies if the gilt settles in the first coupon period).

 $s_1$  = Number of calendar days in the full coupon period in which the <u>issue</u> date occurs.

 $s_2$  = Number of calendar days in the full coupon period after the coupon period in which the <u>issue</u> date occurs.

 $r_1$  = Number of calendar days from the <u>issue</u> date to the next (theoretical) coupon date.

= Number of calendar days from the (theoretical) coupon date after the <u>issue</u> date to the settlement date in the coupon period after the coupon period in which the <u>issue</u> date occurs (this term only applies if the gilt settles in the second coupon period.

RPID = The RPI which fixes the first dividend payment for the gilt i.e. the RPI

scheduled to be published seven months prior to the month of the first dividend payment and relating to the month before that prior month (for example, if the first dividend payment on the gilt will be in November then the RPI which fixes its value is the RPI for March of that year).

RPIB = The base RPI for the gilt i.e. the RPI scheduled to be published seven months prior to the month of issue of the gilt and relating to the month before that prior month (for example, if the gilt is issued in November then its base RPI is the RPI for March of that year).

Where the relevant *RPID* has not been published (at the time of issue) for a long first dividend value to be able to be fixed, the DMO would set a short first dividend on the new stock.

The accrued interest on all gilts is rounded to the nearest penny on the traded nominal amount for calculating settlement proceeds.

## ANNEX: ESTIMATION OF THE NOMINAL VALUES OF FUTURE UNKNOWN CASH FLOWS ON INDEX-LINKED GILTS AND ON FLOATING RATE GILTS

#### Index-linked Gilts

For the purpose of computing the gilt's settlement price, the nominal values of unknown future dividends are estimated as:

$$d_{i+1} = \frac{c}{2} \times \frac{a}{u^i} \quad 1 \le i \le n$$

Where:  $d_{i+1}$  = Dividend due on (i+1)th quasi-coupon date after the settlement date, per £100 nominal of the gilt.

c = Coupon per £100 nominal of the gilt.

 $\pi$  = The assumed annual inflation rate (decimal) = 0.03.

$$u = \left(\frac{1}{1+\pi}\right)^{\frac{1}{2}} = \left(\frac{1}{1.03}\right)^{\frac{1}{2}}$$

RPIB = The base RPI for the gilt i.e. the RPI scheduled to be published seven months prior to the month of issue of the gilt and relating to the month before that prior month (for example, if the gilt is issued in November then its base RPI is the RPI for March of that year).

*RPIL* = The latest published RPI at the time of settlement.

k = Number of months between the month of the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the month of the latest published RPI at the time of settlement. For example, if the RPI for January is the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the latest published RPI at the time of settlement is the RPI for April, then k = 3.

$$a = \frac{RPIL}{RPIB} \cdot u^{\frac{2k}{12}}$$

n = Number of full coupon periods between the next quasi-coupon date and redemption.

In addition, in most cases the RPI determining the redemption payment will not have been published, so that the nominal value of the redemption payment will not be known at the time of settlement. For the purpose of computing the gilt's price, the nominal value of the redemption payment is estimated as:

$$R = 100 \times \frac{a}{u^n}$$

Where: R = Redemption payment per £100 nominal of the gilt.

c = Coupon per £100 nominal of the gilt.

 $\pi$  = The assumed annual inflation rate (decimal) = 0.03.

$$u = \left(\frac{1}{1+\pi}\right)^{\frac{1}{2}} = \left(\frac{1}{1.03}\right)^{\frac{1}{2}}$$

RPIB = The base RPI for the gilt i.e. the RPI scheduled to be published seven months prior to the month of issue of the gilt and relating to the month before that prior month (for example, if the gilt is issued in November then its base RPI is the RPI for March of that year).

*RPIL* = The latest published RPI at the time of settlement.

k = Number of months between the month of the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the month of the latest published RPI at the time of settlement. For example, if the RPI for January is the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the latest published RPI at the time of settlement is the RPI for April, then k = 3.

n = Number of full coupon periods between the next quasi-coupon date and redemption.

All estimated index-linked gilt cash flows are left unrounded for price/yield calculation purposes<sup>9</sup>.

#### Floating Rate Gilts

The value of the next but one and subsequent dividends will not be known until the business day before the day of the preceding dividend payment, but for the purpose of computing the price of the gilt these are estimated as:

$$d_i = \frac{(L+D) \times r_{i-1}}{365} \quad 2 \le i \le n+1$$

Where:  $d_i$  = The dividend on the *i*th quasi-coupon date after the settlement date per £100

nominal of the gilt.

L = The latest value of *Index* (rounded to the nearest 5th decimal place),

used as the reference to set future coupon payments.

D = The discount (negative) or premium (positive) at which the coupon is fixed

relative to *Index* (e.g. for a Floating Rate Gilt of coupon *Index*  $-\frac{1}{8}$ ,  $D = -\frac{1}{8}$ ).

 $r_i$  = Number of calendar days in the *i*th coupon period, where the coupon period

after the coupon period in which the settlement date occurs is defined as the

first coupon period.

n =Number of full coupon periods between the next quasi-coupon date and

redemption.

All estimated Floating Rate Gilt cash flows are left unrounded for price/yield calculation purposes.

#### **NOTES**

- In order to solve some types of equation it is necessary to obtain numerical approximations to the roots using an iterative process. An iterative process starts with an approximation x<sub>0</sub> to a root λ from which another approximation x<sub>1</sub> is obtained, and then another approximation x<sub>2</sub>, and so on. For an effective process (for a particular root) the successive values (or iterates) x<sub>1</sub>, x<sub>2</sub>, x<sub>3</sub>,... should become progressively closer to the root λ. The process is continued until an approximation of the required accuracy is obtained.
- 2. See the section on double-dated and undated gilts for how to work out the assumed redemption date.
- 3. The dirty price of a gilt is its total settlement price which includes accrued interest.
- 4. If this has not yet been published by the authorities, see the Annex for how to estimate it.
- 5. The two floating rate gilts that have been issued (now redeemed) were set relative to the index LIBID, as measured by the Bank of England.
- 6. In practice only one cash flow will have been fixed on any settlement date.
- 7. For analytical purposes, it may be desirable to use a more refined estimate of m, for example, by interpolating between two Index rates with maturities either side of q days.
- 8. The ex-dividend date for all gilts except 3 1/2% War Loan is currently the date seven business days before the dividend date; for 3 1/2% War Loan it is the date ten business days before the dividend date. (The last Floating Rate Gilt, which redeemed in July 2001, had no ex-dividend date.)
- 9. Actual cash flows on index-linked gilts are rounded as follows per £100 nominal: (a) 2% IL 2006 and 2 1/2% IL 2011: rounded down to 2 decimal places; (b) 2 1/2% IL 2003, 4 3/8% IL 2004, 2 1/2% IL 2009, 2 1/2% IL 2013, 2 1/2% IL 2016, 2 1/2% IL 2020, 2 1/2% IL 2024 and

1/8% IL 2030: rounded down to 4 decimal places; (c) all other index-linked gilts (i.e. those first issued after January 2002): rounded to the nearest  $6^{th}$  decimal place.