BANK OF ENGLAND FORMULAE FOR CALCULATING GILT PRICES FROM YIELDS

This paper sets out the formulae that will be used in the London Stock Exchange's regulatory guide to the gilts market for calculating gilt prices from gross redemption yields, thus allowing a formal settlement convention to be applied to a trade conducted on a yield basis. In the event that the formulae are to be used to derive yields from prices it is not possible (in most cases) to solve for yield in terms of price algebraically, and so some form of numerical technique[1] must be used if, given a price, a value for the redemption yield is required.

The first section of the paper states the formulae that will be used; these are split into the different classes of gilt (new formulae for new instruments will be added to the guide as and when required). For the purpose of this paper, "cash flows" refer to cash flows receivable by the buyer of the gilt. Also, "nearest rounding" to, say, six decimal places means round the sixth decimal place up by one if the seventh decimal place is 5 or above, and then truncate at the sixth decimal place.

Compounding will occur on quasi-coupon dates. Quasi-coupon dates are the dates on the semi-annual cycle (or quarterly cycle, for quarterly paying gilts) defined by the maturity date, irrespective of whether cash flows occur on those dates (examples of quasi-coupon dates on which cash flows would not occur include the first quasi-coupon date of a new issue having a long first dividend period; the next quasi-coupon date of a gilt settling in its ex-dividend period; and most quasi-coupon dates of a strip). The quasi-coupon dates for undated gilts are defined by their regular coupon cycle. A full (quasi-) coupon period is defined as the period between two consecutive quasi-coupon dates. For example, a gilt settling on its issue date (assuming this is not also a quasi-coupon date) will have a quasi-coupon period which starts on the quasi-coupon date prior to the issue date and ends on the first quasi-coupon date following the issue date. Cash flows and quasi-coupon dates which are due to occur on non-business days are not adjusted.

This means that cash flows which occur off quasi-coupon dates (such as some early redemption payments - hypothetical or actual - on double-dated gilts) will have an additional fractional period associated with their discounting process to allow for discounting back (ie towards the settlement date) by a fractional period to the quasi-coupon date immediately prior to their occurrence, before being discounted back to the settlement date.

Currently there is no formal convention on the rounding of gilt prices computed from yields. Since the strips market will work on a convention of rounding to the nearest 6th decimal place per £100, it is proposed that all dirty prices computed from yields should be rounded to the nearest 6th decimal place per £100. In addition, the price/yield formulae discount all cash flows on the quasi-coupon cycle using the 'actual/actual' daycount convention: this is consistent with the agreed market consensus for discounting the cash flow from a strip.

Following market consultation, the inflation assumption that will be used in the formulae for index-linked gilts is 3% per annum. This will be reviewed by the authorities as and when a majority of market participants judge that a review is necessary.

The second section provides formulae for the calculation of accrued interest: these currently follow the 'actual/365' daycount convention. In February 1997 the Bank of England consulted the market on whether to change the daycount convention used for the calculation of accrued interest from 'actual/365' to 'actual/actual'. The majority of the respondents indicated a wish to change the convention to 'actual/actual'. The Bank proposes that the change will be made next year. Its implementation date will be determined in consultation with market participants as soon as possible, and will take account of the implications for firms' systems and for the specification of LIFFE long gilt futures contracts; the change will not be implemented before July 1998.

An annex describing the procedure for estimating the nominal values of unknown future cash flows on index-linked gilts and on Floating Rate Gilts can be found at the end of the paper.

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SECTION ONE: PRICE/YIELD FORMULAE

Conventional Gilts; Double-dated and Undated Gilts with Assumed (or Actual) Redemption on a Quasi-Coupon Date[2]

The formula for calculating the price from the yield is given by:

$$P = v^{\frac{r}{s}}\left(d_1 + d_2 v + \frac{cv^2}{f(1-v)}(1-v^{n-1}) + 100v^n\right) - \sum_{j=1}^{m} pc_j \cdot v^{\frac{tc_{j,1}}{s}+\frac{tc_{j,2}}{s'}} \qquad \text{for } n \geq 1$$

Where:

P = Dirty price per £100 nominal of the gilt.[3]

d_1 = Cash flow due on next quasi-coupon date, per £100 nominal of the gilt (may be zero if the gilt has a long first dividend period or if the gilt settles in its ex-dividend period; or may be greater or less than $\frac{c}{2}$ during long or short first dividend periods respectively).

d_2 = Cash flow due on next but one quasi-coupon date, per £100 nominal of the gilt (may be greater than $\frac{c}{2}$ during long first dividend periods).

c = Coupon per £100 nominal of the gilt.

y = Nominal redemption yield (decimal) ie if the yield is 8% then y =0.08.

f = Number of coupons payable on the gilt per year (f will be equal to 2 or 4).

v = $\frac{1}{1+\frac{y}{f}}$

r = Number of calendar days from the settlement date to the next quasi-coupon date.

s = Number of calendar days in the full coupon period in which the settlement date occurs (ie between the prior quasi-coupon date and the following quasi-coupon date).

n = Number of full coupon periods between the next quasi-coupon date and redemption.

m = Number of remaining part-payments (only applies to partly-paid gilts).

pc_j = Size of the *j*th remaining part-payment per £100 nominal.

s' = Number of calendar days in the full coupon period after the coupon period in which the settlement date occurs.

$tc_{j,1}$ = Number of calendar days from the settlement date to the next quasi-coupon date (or the date of the *j*th remaining part-payment if this is earlier than the next quasi-coupon date).

$tc_{j,2}$ = Number of calendar days from the next quasi-coupon date to the date of the *j*th remaining part-payment if this is later than the next quasi-coupon date (otherwise $tc_{j,2}$ is equal to zero).

The price is rounded to the nearest 6th decimal place.

For $n = 0$, the equation reduces to

$$P = v^{\frac{r}{s}}\left(d_1 + 100\right)$$

<u>Index-linked Gilts</u>

(1) Not all the nominal values of future cash flows are fixed

Case 1: *Two or more cash flows remaining*

The formula for calculating the price from the yield is given by:

$$P = \left[d_1 + d_2(uw) + \frac{acw^2}{2(1-w)}(1-w^{n-1})\right](uw)^{\frac{r}{s}} + 100au^{\frac{r}{s}}w^{\frac{r}{s}+n} - \sum_{j=1}^{m} pc_j(uw)^{\frac{tc_{j,1}}{s}+\frac{tc_{j,2}}{s'}} \quad , \; n \geq 1$$

Where: P = Dirty price per £100 nominal of the gilt[3].

d_1 = Cash flow due on next quasi-coupon date, per £100 nominal of the gilt (may be zero if the gilt has a long first dividend period or if the gilt settles in its ex-dividend period; or may be greater or less than $\frac{c}{2}$ during long or short first dividend periods respectively).

d_2 = Cash flow due on next but one quasi-coupon date, per £100 nominal of the gilt (may be greater than $\frac{c}{2}$ times the RPI Ratio during long first dividend periods)[4].

c = (Real) coupon per £100 nominal.

r = Number of calendar days from the settlement date to the next quasi-coupon date.

s = Number of calendar days in the full coupon period in which the settlement date occurs (ie between the prior quasi-coupon date and the following quasi-coupon date).

ρ = Semi-annually compounded real redemption yield (decimal) i.e. if the real yield is 3% then ρ = 0.03.

w = $\dfrac{1}{1+\frac{\rho}{2}}$

π = The assumed annual inflation rate (decimal) = 0.03.

u = $\left(\dfrac{1}{1+\pi}\right)^{\frac{1}{2}} = \left(\dfrac{1}{1.03}\right)^{\frac{1}{2}}$

n = Number of full coupon periods between the next quasi-coupon date and redemption.

m = Number of remaining part-payments (only applies to partly-paid gilts).

pc_j = Size of the *j*th remaining part-payment per £100 nominal.

s' = Number of calendar days in the full coupon period after the coupon period in which the settlement date occurs.

$tc_{j,1}$ = Number of calendar days from the settlement date to the next quasi-coupon date (or the date of the *j*th remaining part-payment if this is earlier than the next quasi-coupon date).

$tc_{j,2}$ = Number of calendar days from the next quasi-coupon date to the date of the *j*th remaining part-payment if this is later than the next quasi-coupon date (otherwise $tc_{j,2}$ is equal to zero).

RPIB = The base RPI for the stock ie the RPI scheduled to be published seven months prior to the month of issue of the gilt and relating to the month before that prior month (for example, if the stock is issued in November then its base RPI is the RPI for March of that year).

RPIL = The latest published RPI at the time of settlement.

k = Number of months between the month of the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the month of the latest published RPI at the time of settlement. For example, if the RPI for January is the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the latest published RPI at the time of settlement is the RPI for April, then k = 3.

a = $\frac{RPIL}{RPIB} \cdot u^{\frac{2k}{12}}$

The price is rounded to the nearest 6th decimal place.

Case 2: One cash flow remaining (ie the final dividend and redemption payment)

If the RPI determining the redemption value is published after the stock goes ex-dividend for the penultimate time, the price/yield formula is defined as:

$$P = \left(100 + \frac{c}{2}\right) \cdot a(uw)^{\frac{r}{s}+\alpha}$$

Where: P = Dirty price per £100 nominal of the gilt[3].

c = (Real) coupon per £100 nominal.

ρ = Real redemption yield (decimal) ie if the yield is 3% then ρ =0.03.

w = $\frac{1}{1+\frac{\rho}{2}}$

π = The assumed annual inflation rate (decimal) = 0.03.

$u \quad = \left(\frac{1}{1+\pi}\right)^{\frac{1}{2}} = \left(\frac{1}{1.03}\right)^{\frac{1}{2}}$

r = Number of calendar days from the settlement date to the next quasi-coupon date.

s = Number of calendar days in the full coupon period in which the settlement date occurs (ie between the prior quasi-coupon date and the following quasi-coupon date).

$$\alpha = \begin{cases} 1 & \text{if the gilt is settling in its penultimate ex-dividend period} \\ 0 & \text{if the gilt is settling after its penultimate quasi-coupon date} \end{cases}$$

RPIB = The base RPI for the stock ie the RPI scheduled to be published seven months prior to the month of issue of the gilt and relating to the month before that prior month (for example, if the stock is issued in November then its base RPI is the RPI for March of that year).

RPIL = The latest published RPI at the time of settlement.

k = Number of months between the month of the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the month of the latest published RPI at the time of settlement. For example, if the RPI for January is the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the latest published RPI at the time of settlement is the RPI for April, then k = 3.

$$a = \frac{RPIL}{RPIB} \cdot u^{\frac{2k}{12}}$$

The price is rounded to the nearest 6th decimal place.

In this special case, we can solve algebraically for yield in terms of price:

$$\rho = 2 \cdot \left(u \cdot \left(\frac{P}{\left(100 + \frac{c}{2}\right) \cdot a} \right)^{-\frac{s}{r+\alpha s}} - 1 \right)$$

(2) Nominal values of all future cash flows are fixed

Case 1: Index-linked stocks that have passed both their penultimate ex-dividend date and the date on which the RPI determining the final redemption payment is published provide a known cash flow on just one remaining date. The price/yield formula in this case is:

$$P = v^{\frac{r}{s}+\alpha}\left(d_{LAST} + R\right)$$

Where: P = Dirty price per £100 nominal of the gilt[3].

d_{LAST} = Final dividend payment per £100 nominal of the gilt.

R = Final redemption payment per £100 nominal of the gilt.

y = Semi-annually compounded nominal redemption yield (decimal) ie if the yield is 8% then y=0.08.

$v = \frac{1}{1+\frac{y}{2}}$

r = Number of calendar days from the settlement date to the next quasi-coupon date.

s = Number of calendar days in the full coupon period in which the settlement date occurs (ie between the prior quasi-coupon date and the following quasi-coupon date).

$$\alpha = \begin{cases} 1 & \text{if the gilt is settling in its penultimate ex-dividend period} \\ 0 & \text{if the gilt is settling after its penultimate quasi-coupon date} \end{cases}$$

The price is rounded to the nearest 6th decimal place.

In this special case, we can solve algebraically for yield in terms of price:

$$y = 2 \cdot \left[\left(\frac{P}{d_{LAST} + R} \right)^{-\frac{s}{r+\alpha s}} - 1 \right]$$

Case 2: When valuing index-linked stocks on days between the publication date of the RPI determining the redemption payment and the penultimate ex-dividend date (assuming that the RPI determining the redemption value is published before the stock goes ex-dividend for the penultimate time), the price/yield formula is defined as:

$$P = \left(d_{PEN} + \left(d_{LAST} + R \right) \cdot v \right) v^{\frac{r}{s}}$$

Where: P = Dirty price per £100 nominal of the gilt[3].

d_{PEN} = Penultimate dividend payment per £100 nominal of the gilt.

d_{LAST} = Final dividend payment per £100 nominal of the gilt.

y = Semi-annually compounded nominal redemption yield (decimal) ie if the yield is 8% then y=0.08.

$v = \frac{1}{1+\frac{y}{2}}$

R = Redemption payment per £100 nominal of the gilt.

r = Number of calendar days from the settlement date to the next quasi-coupon date.

s = Number of calendar days in the full coupon period in which the settlement date occurs (ie between the prior quasi-coupon date and the following quasi-coupon date).

The price is rounded to the nearest 6th decimal place.

Double-dated Gilts

A double-dated gilt has a final maturity date and in addition an earlier maturity date, with Her Majesty's Treasury having the right to redeem the gilt on any day between these two dates, provided that the relevant notice is given (usually 3 months). In order to calculate the redemption yield for such gilts it is necessary to make some assumption about when the gilt will be redeemed (where a specific redemption date has not yet been announced by the authorities). The convention used in these formulae is referred to as the *yield/coupon rule*:

Case 1: *The settlement date is more than x months before the first date in the redeemable band (where x is the period of notice required to be given to call the gilt as specified in its prospectus - usually 3 months).* Then if the nominal redemption yield y is greater than or equal to the coupon, the latest redemption date in the redeemable band is assumed; otherwise the earliest redemption date in the redeemable band is assumed.

Case 2: The settlement date is either less than x months before the first date in the redeemable band (where x is the period of notice required to be given to call the gilt as specified in its prospectus - usually 3 months), or the settlement date is in the redeemable band. Then if notice has not yet been given by the authorities that the gilt will be redeemed early, the latest redemption date in the redeemable band is assumed (irrespective of whether the nominal redemption yield y is greater than or less than the coupon).

Having made such an assumption about the redemption date, if this is on a quasi-coupon date the formula for conventional gilts should be used; if this is on a date which is not a quasi-coupon date, the following formula should be used:

$$P = v^{\frac{r}{s}} \left(d_1 + d_2 v + \frac{cv^2}{2(1-v)}(1 - v^{n-1}) + (100 + d_f) \cdot v^n \cdot v^{\frac{t}{u}} \right) \quad \text{for } n \geq 1$$

Where: P = Dirty price per £100 nominal of the gilt[3].

d_1 = Cash flow due on next quasi-coupon date, per £100 nominal of the gilt (may be zero if the gilt has a long first dividend period or if the gilt settles in its ex-dividend period; or may be greater or less than $\frac{c}{2}$ during long or short first dividend periods respectively).

d_2 = Cash flow due on next but one quasi-coupon date, per £100 nominal of the gilt (may be greater than $\frac{c}{2}$ during long first dividend periods).

d_f = Partial coupon due on off quasi-coupon redemption date, per £100 nominal of the gilt.

c = Coupon per £100 nominal of the gilt.

y = Semi-annually compounded nominal redemption yield (decimal) ie if the yield is 8% then y=0.08.

v = $\frac{1}{1+\frac{y}{2}}$

r = Number of calendar days from the settlement date to the next quasi-coupon date.

s = Number of calendar days in the full coupon period in which the settlement date occurs (ie between the prior quasi-coupon date and the following quasi-coupon date).

t = Number of calendar days from the redemption date to the preceding quasi-coupon date.

u = Number of calendar days in the full coupon period in which the redemption

date occurs.

n = Number of full coupon periods between the next quasi-coupon date and redemption.

For $n = 0$, the equation reduces to

$$P = v^{\frac{r}{s}}\left(d_1 + \left(100 + d_f\right)\cdot v^{\frac{t}{u}}\right)$$

Undated Gilts

All current undated gilts in issue have a date after which they can be redeemed (for example, 3 1/2% War Loan is dated '1952 or after'). In order to calculate the redemption yield for such gilts it is necessary to make some assumption about when the gilt will be redeemed (where a specific redemption date has not yet been announced by the authorities). Again the *yield/coupon rule* is used to determine the assumed redemption date:

Case 1: *The settlement date is more than x months before the first date in the redeemable band (where x is the period of notice required to be given to call the gilt as specified in its prospectus - usually 3 months).* Then if the nominal redemption yield y is greater than or equal to the coupon, it is assumed the gilt will not be called and the infinite cash flow formula should be used (see below); otherwise the earliest redemption date in the redeemable band is assumed.

Case 2: The settlement date is either less than x months before the first date in the redeemable band (where x is the period of notice required to be given to call the gilt as specified in its prospectus - usually 3 months), or the settlement date is in the redeemable band. Then if notice has not yet been given by the authorities that the gilt will be redeemed early, it is assumed that the gilt will not be redeemed and the infinite cash flow formula should be used (see below), irrespective of whether the nominal redemption yield y is greater than or less than the coupon.

For an assumed or actual early redemption date, if this is on a quasi-coupon date the formula for conventional gilts should be used; if this is on a date which is not a quasi-coupon date, the following formula should be used:

$$P = v^{\frac{r}{s}}\left(d_1 + d_2 v + \frac{cv^2}{f(1-v)}\left(1-v^{n-1}\right) + \left(100 + d_f\right)\cdot v^n \cdot v^{\frac{t}{u}}\right) \qquad \text{for } n \geq 1$$

Where: P = Dirty price per £100 nominal of the gilt[3].

d_1 = Cash flow due on next quasi-coupon date, per £100 nominal of the gilt (may be zero if the gilt has a long first dividend period or if the gilt settles in its ex-dividend period; or may be greater or less than $\frac{c}{2}$ during long or short first dividend periods respectively).

d_2 = Cash flow due on next but one quasi-coupon date, per £100 nominal of the gilt (may be greater than $\frac{c}{2}$ during long first dividend periods).

d_f = Partial coupon due on off quasi-coupon redemption date, per £100 nominal of the gilt.

c = Coupon per £100 nominal of the gilt.

y = Nominal redemption yield (decimal) ie if the yield is 8% then y=0.08.

f = Number of coupons payable on the gilt per year (f will be equal to 2 or 4).

v = $\frac{1}{1+\frac{y}{f}}$

r = Number of calendar days from the settlement date to the next quasi-coupon date.

s = Number of calendar days in the full coupon period in which the settlement date occurs (ie between the prior quasi-coupon date and the following quasi-coupon date).

t = Number of calendar days from the redemption date to the preceding quasi-coupon date.

u = Number of calendar days in the full coupon period in which the redemption date occurs.

n = Number of full coupon periods between the next quasi-coupon date and redemption.

The price is rounded to the nearest 6th decimal place.

For $n = 0$, the equation reduces to

$$P = v^{\frac{r}{s}}\left(d_1 + (100 + d_f)\cdot v^{\frac{t}{u}}\right)$$

Infinite cash flow method: For an infinite set of cash flows (ie where it is assumed that the gilt will not be redeemed early) we use the formula for a conventional gilt and take P to be the limit of the sum of the discounted cash flows as *n* (the number of full coupon periods between the next quasi-coupon date and redemption) tends to infinity. Since $|v|<1$, this limit exists and is equal to

$$P = v^{\frac{r}{s}}\left(d_1 + d_2 v + \frac{cv^2}{f(1-v)}\right)$$

Where: P = Dirty price per £100 nominal of the gilt[3].

d_1 = Cash flow due on next quasi-coupon date, per £100 nominal of the gilt (may be zero if the gilt has a long first dividend period or if the gilt settles in its ex-dividend period; or may be greater or less than $\frac{c}{2}$ during long or short first dividend periods respectively).

d_2 = Cash flow due on next but one quasi-coupon date, per £100 nominal of the gilt (may be greater than $\frac{c}{2}$ during long first dividend periods).

c = Coupon per £100 nominal of the gilt.

y = Nominal redemption yield (decimal) ie if the yield is 8% then y =0.08.

f = Number of coupons payable on the gilt per year (f will be equal to 2 or 4).

v = $\frac{1}{1+\frac{y}{f}}$

r = Number of calendar days from the settlement date to the next quasi-coupon date.

s = Number of calendar days in the full coupon period in which the settlement date occurs (ie between the prior quasi-coupon date and the following quasi-coupon date).

The price is rounded to the nearest 6th decimal place.

Floating Rate Gilts

Unlike conventional gilts, the coupons on Floating Rate Gilts are not fixed but vary in line with some index such as LIBID[5]. Instead of computing a redemption yield for such gilts we calculate the discount at which they trade relative to LIBID. Given a discount margin y the price is computed using the formula below. It is not possible to solve for the discount margin in terms of price algebraically, and so some form of numerical technique[1] must be used if, given a price, a value for the discount margin relative to LIBID is required.

$$P = \frac{d_1}{\left(1+\frac{(m+y)q}{36500}\right)} + \frac{1}{\left(1+\frac{(m+y)q}{36500}\right)}\left[\sum_{k=1}^{n}\frac{\frac{r_{k+1}\times(L+D)}{365}}{\prod_{j=1}^{k}\left(1+\frac{r_{j+1}\times\left(L+y\right)}{36500}\right)} + \frac{100}{\prod_{j=1}^{n}\left(1+\frac{r_{j+1}\times\left(L+y\right)}{36500}\right)}\right]$$

Where:

- P = Dirty price per £100 nominal of the gilt[3].
- d_1 = Next receivable dividend per £100 nominal of the gilt[4].
- q = Number of calendar days from the settlement date to the next dividend date.
- r_{k+1} = Number of calendar days in the (k+1)th dividend period.
- n = Number of dividends due on the gilt after the next receivable dividend.
- L = The latest value of LIBID (rounded to the nearest 5th decimal place) that is used as the reference to set future coupon payments, quoted as a percentage.
- m = The latest q-day LIBID rate (rounded to the nearest 5th decimal place), quoted as a percentage.
- D = The discount (negative) or premium (positive) at which the coupon is fixed relative to LIBID (eg for a Floating Rate Gilt of coupon LIBID $-\frac{1}{8}$, $D=-\frac{1}{8}$).
- y = The discount (negative) or premium (positive) at which the gilt trades relative to LIBID, quoted as a percentage.

The price is rounded to the nearest 6th decimal place.

<u>Strips</u>

In February 1997 the Bank published a consultative paper seeking views on what standardised formula for computing market prices from gross redemption yields should be adopted to allow gilt strips to trade on a yield basis. The result of the consultation was indicated by Press Notices on 30 May 1997 and 12 June 1997. The market consensus was that the following method was the most suitable for strips:

$$P = \frac{100}{\left(1+\frac{y}{2}\right)^{\frac{r}{s}+n}}$$

Where: P = Dirty price per £100 nominal of the strip[3].

y = Strip gross redemption yield (decimal) ie if the yield is 8% then $y = 0.08$.

r = Number of calendar days from the settlement date to the next quasi-coupon date.

s = Number of calendar days in the quasi-coupon period in which the settlement date occurs (ie between the prior quasi-coupon date and the following quasi-coupon date).

n = Number of full coupon periods between the next quasi-coupon date and redemption.

The price is rounded to the nearest 6th decimal place.

In this special case, we can solve algebraically for yield in terms of price:

$$y = 2 \cdot \left[\left(\frac{100}{P}\right)^{\frac{s}{r+ns}} - 1\right]$$

SECTION TWO: CALCULATION OF ACCRUED INTEREST

While coupon payments on individual gilts are usually made only twice a year, gilts can be traded on any working day. Whenever a gilt changes hands on a day that is not a coupon payment date, the valuation of the gilt will reflect the proximity of the next coupon payment date. This is effected by the payment of accrued interest to compensate the seller for the period since the last coupon payment date during which the seller has held the gilt but for which he receives no coupon payment. The accrued interest is computed as follows[6]:

(1) Fully-paid gilts

(i) All gilts excluding Floating Rate Gilts and quarterly paying undated gilts[7]

$$AI = \begin{cases} \dfrac{t_1}{182.5} \cdot d_1 & \text{if the settlement date occurs on or before the ex-dividend date} \\ \dfrac{-t_2}{182.5} \cdot d_1 & \text{if the settlement date occurs after the ex-dividend date} \end{cases}$$

Where: AI = Accrued interest per £100 nominal of the gilt.

d_1 = Next dividend per £100 nominal of the gilt.

t_1 = Number of calendar days from the last dividend date[8] to the settlement date.

t_2 = Number of calendar days from the settlement date to the next dividend date.

(ii) Floating Rate Gilts which have an ex-dividend period (for example, Floating Rate Treasury Stock 1999)

$$AI = \begin{cases} \dfrac{t}{s} \cdot d_1 & \text{if the settlement date occurs on or before the ex-dividend date} \\ \left(\dfrac{t}{s} - 1\right) \cdot d_1 & \text{if the settlement date occurs after the ex-dividend date} \end{cases}$$

Where: AI = Accrued interest per £100 nominal of the gilt.

d_1 = Next dividend per £100 nominal of the gilt.

t = Number of calendar days from the last dividend date[8] to the settlement date.

s = Number of calendar days in the full coupon period in which the settlement date occurs (ie between the prior quasi-coupon date and the following quasi-coupon date).

(iii) Floating Rate Gilts which do not have an ex-dividend period (for example, Floating Rate Treasury Stock 2001)

$$AI = \frac{t}{s} \cdot d_1$$

Where: AI = Accrued interest per £100 nominal of the gilt.

d_1 = Next dividend per £100 nominal of the gilt.

t = Number of calendar days from the last dividend date[8] to the settlement date.

s = Number of calendar days in the full coupon period in which the settlement date occurs (ie between the prior quasi-coupon date and the following quasi-coupon date).

(iv) Quarterly paying gilts (excluding Floating Rate Gilts)

$$AI = \begin{cases} \dfrac{t_1}{91.25} \cdot d_1 & \text{if the settlement date occurs on or before the ex-dividend date} \\ \dfrac{-t_2}{91.25} \cdot d_1 & \text{if the settlement date occurs after the ex-dividend date} \end{cases}$$

Where: AI = Accrued interest per £100 nominal of the gilt.

d_1 = Next dividend per £100 nominal of the gilt.

t_1 = Number of calendar days from the last dividend date[8] to the settlement date.

t_2 = Number of calendar days from the settlement date to the next dividend date.

(2) Partly-paid gilts

In the case of partly-paid gilts, interest accrues on the amount of the gilt paid for as a proportion of the fully-paid price (or the minimum price when one is given). Gilts issued by auction have been deemed to be sold at £100.

$$AI = \sum_{j=1}^{m_a} \frac{sp_j}{p_{TOT}} \cdot \frac{t_j}{182.5} \cdot \frac{c}{2}$$

Where:

AI = Accrued interest per £100 nominal of the gilt.

sp_j = Sum of the part-payments made between the issue date and the *j*th part-payment (sp_1 is assumed to be the payment made at issue; for gilts issued by auction, sp_1 is defined to be £100 minus the sum of future part-payments to be made).

p_{TOT} = The minimum price of the gilt when one is given. Otherwise the full issue price (including the sum of any part-payments) of the gilt (gilts issued by auction have been deemed to be sold at £100).

m_a = Number of part-payments from the issue date to the settlement date (the payment made at issue is included in this count).

t_j = Number of calendar days between the (*j*-1)th part-payment (or the issue date when *j* = 1) and the (*j*+1)th part-payment (or the settlement date if the *j*th part-payment is the last one before the settlement date).

c = Coupon per £100 nominal of the gilt.

The accrued interest on all gilts is rounded to the nearest 5th decimal place.

ANNEX: ESTIMATION OF THE NOMINAL VALUES OF FUTURE UNKNOWN CASH FLOWS ON INDEX-LINKED GILTS AND ON FLOATING RATE GILTS

Index-linked Gilts

For the purpose of computing the gilt's settlement price, the nominal values of unknown future dividends are estimated as:

$$d_{i+1} = \frac{c}{2} \times \frac{a}{u^i} \qquad 1 \le i \le n$$

Where: d_{i+1} = Dividend due on (i+1)th quasi-coupon date after the settlement date, per £100 nominal of the gilt.

c = Coupon per £100 nominal of the gilt.

π = The assumed annual inflation rate (decimal) = 0.03.

$$u = \left(\frac{1}{1+\pi}\right)^{\frac{1}{2}} = \left(\frac{1}{1.03}\right)^{\frac{1}{2}}$$

$RPIB$ = The base RPI for the gilt ie the RPI scheduled to be published seven months prior to the month of issue of the gilt and relating to the month before that prior month (for example, if the gilt is issued in November then its base RPI is the RPI for March of that year).

$RPIL$ = The latest published RPI at the time of settlement.

k = Number of months between the month of the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the month of the latest published RPI at the time of settlement. For example, if the RPI for January is the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the latest published RPI at the time of settlement is the RPI for April, then k = 3.

$$a = \frac{RPIL}{RPIB} \cdot u^{\frac{2k}{12}}$$

n = Number of full coupon periods between the next quasi-coupon date and redemption.

In addition, in most cases the RPI determining the redemption payment will not have been published, so that the nominal value of the redemption payment will not be known at the time of settlement. For the purpose of computing the gilt's price, the nominal value of the redemption payment is estimated as:

$$R = 100 \times \frac{a}{u^n}$$

Where: R = Redemption payment per £100 nominal of the gilt.

c = Coupon per £100 nominal of the gilt.

π = The assumed annual inflation rate (decimal) = 0.03.

u = $\left(\frac{1}{1+\pi}\right)^{\frac{1}{2}} = \left(\frac{1}{1.03}\right)^{\frac{1}{2}}$

$RPIB$ = The base RPI for the gilt ie the RPI scheduled to be published seven months prior to the month of issue of the gilt and relating to the month before that prior month (for example, if the gilt is issued in November then its base RPI is the RPI for March of that year).

$RPIL$ = The latest published RPI at the time of settlement.

k = Number of months between the month of the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the month of the latest published RPI at the time of settlement. For example, if the RPI for January is the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the latest published RPI at the time of settlement is the RPI for April, then k = 3.

n = Number of full coupon periods between the next quasi-coupon date and redemption.

Floating Rate Gilts

The value of the next but one and subsequent dividends will not be known until the business day before the day of the preceding dividend payment (or on the day of the preceding dividend payment for the Floating Rate Treasury Stock 1999), but for the purpose of computing the price of the gilt these are estimated as:

$$d_{i+1} = \frac{(L + D) \times r_{i+1}}{365} \qquad 1 \le i \le n$$

Where: d_{i+1} = The ith dividend after the next receivable dividend per £100 nominal of the gilt.

L = The latest value of LIBID (rounded to the nearest 5th decimal place) that is used as the reference to set future coupon payments[9].

D = The discount (negative) or premium (positive) at which the coupon is fixed relative to LIBID (eg for a Floating Rate Gilt of coupon LIBID $-\frac{1}{8}$, $D = -\frac{1}{8}$).

r_{i+1} = Number of calendar days in the (i+1)th dividend period.

n = Number of remaining dividends on the gilt after the next receivable dividend.

NOTES

1. In order to solve some types of equation it is necessary to obtain numerical approximations to the roots using an iterative process. An iterative process starts with an approximation x_0 to a root λ from which another approximation x_1 is obtained, and then another approximation x_2, and so on. For an effective process (for a particular root) the successive values (or iterates) x_1, x_2, x_3,... should become progressively closer to the root λ. The process is continued until an approximation of the required accuracy is obtained.

2. See the section on double-dated and undated gilts for how to work out the assumed redemption date.

3. The dirty price of a gilt is its total price which includes accrued interest but excludes any part-payments outstanding on the settlement date.

4. If this has not yet been published by the authorities, see the Annex for how to estimate it.

5. The two current floating rate gilts are set relative to the index LIBID, as measured by the Bank of England. Were new floating rate gilts to be set relative to a different index, this index would replace LIBID in the formula.

6. The ex-dividend date for all gilts except 3 1/2% War Loan is the date seven working days before the dividend date; for 3 1/2% War Loan it is the date ten working days before the dividend date. The Bank of England consulted the market on whether the ex-dividend period for gilts held in the Central Gilts Office (CGO) should be abolished; and at the same time whether for gilts traded between CGO members and gilt holders outside CGO or holders on the National Savings Stock Register the ex-dividend period should be reduced from 7 to 5 working days (10 to 8 for War Loan). The Bank announced in May 1997 that no decision has yet been made on whether to proceed with these changes, which would require secondary legislation and systems changes at the National Savings Stock Register and at the Bank's Registrar's Department. The implementation date for any change would take into account the implications for firms' systems and for the specification of LIFFE long gilt contracts; any changes would not be implemented before July 1998.

7. These are currently 2 1/2% Consolidated Stock, 2 1/2% Annuities and 2 3/4% Annuities.

8. Or the issue date if d_1 is the first ever dividend.

9. The value of y (the discount margin) is not very sensitive to the estimate of the index value which is chosen.