A simulation model for the analysis of the UK's sovereign debt strategy

Andreas Pick and Myrvin Anthony*+

Abstract

This paper describes a simulation model that staff at the United Kingdom Debt Management Office are developing that may be used in the future for the analysis of the composition of the UK's government bond issuance. The model consists of a trend-deviating five-equation macroeconomic model, Nelson-Siegel type yield curve models for conventional bonds and for inflation-linked bonds and a debt issuance engine. Applying different issuance strategies to the model delivers debt cost distributions that can be analysed for their average cost and risk characteristics. Illustrative issuance strategies are compared on the basis of their average cost and risk properties in order to show how the simulation model works.

1 Introduction

The UK Government borrows funds to finance the excess of cash payments over receipts, to pay interest on outstanding debt and to refinance maturing debt. The Government issues debt instruments in order to raise the cash it wishes to borrow. Currently, government debt instruments are issued with maturities ranging from one month (for T-bills) and 50 years (for bonds), and with interest payment (on bonds) that is both fixed in nominal terms – conventional gilts and linked to inflation[1] – inflation-linked gilts.

The Government can combine these debt instruments in a number of ways to meet its borrowing requirement, but ultimately it has to decide on what it deems to be the best way to borrow these funds. From the Government's fiscal perspective, it would like to borrow funds as cheaply as possible in order to keep down its debt costs and

* Andreas Pick did the main development of the simulation model while he was an economist at the UK Debt Management Office. He also wrote a preliminary paper, which sets out the model and on which this paper draws. We would like to thank Andrew Gurney, Shayne Paterson, Roger Whittaker, Peter Andrews, John Campbell, Yves Nosbusch, Andrew Scott, Shaun Vahey, David Bolder and participants at seminars at the Bank of England and HM Treasury for helpful comments. The views expressed in the paper are the authors' and do not necessarily represent those of the DMO or HM Treasury.

+ Corresponding author. *Email address*: myrvin.anthony@dmo.gsi.gov.uk

[1] As measured by the Retail Prices Index (RPI).

ultimately the cost to the taxpayer. Another consideration for the Government is that the cost associated with a given borrowing strategy should not be too volatile nor expose the Government to unexpected and large increases in debt costs nor should it pose a threat to the attainment of the Government's overall fiscal goals. Hence, what borrowing strategy the Government chooses depends ultimately on these cost and risk considerations.

The consideration of the cost-risk trade-off of borrowing strategies is an important feature of debt management in the UK, as is reflected in the Government's debt management objective. The Government's debt management policy objective is:

> "*to minimise, over the long term, the costs of meeting the Government's financing needs, taking into account risk, whilst ensuring that debt management policy is consistent with the aims of monetary policy*". [2]

Given this debt management objective, the DMO is developing a stochastic simulation model that it may in future use to analyse quantitatively the expected cost and risk of various issuance strategies. This paper describes the key features of this model and provides some illustrative results.

As the simulation model represents work-in-progress it is not presently being used to inform HM Treasury's decisions about the structure of the debt portfolio and the composition of the annual gilt issuance programme set out in the DMO's financing remit each year. Therefore this paper does not describe the current issuance strategy of the Government nor does it define a preferred or optimal issuance strategy for the Government. In fact, on its own this simulation model cannot determine what the Government's preferred debt issuance strategy should be. That can only be determined on the basis of information about the Government's cost-risk trade-off preferences and a consideration of the others factors that the UK authorities examine when choosing a given long-term borrowing strategy. Further, the paper does not express any views about the current stance of the Government's debt management policy nor its likely course in the future. The paper emphasises the methodological framework of the simulation model and shows how one can employ this framework to compare issuance strategies. As will be discussed later, one limitation of the simulation model is that it does not allow for any changes in the relative supply of bonds to influence their yields. One consequence of this limitation is that the model throws up corner solutions, which are unlikely to be pursued in practice.

Given that debt management objectives are similar in many OECD countries, in recent years, other OECD debt managers have also developed and used stochastic simulation modelling in their debt management processes and a small body of research has developed that attempts to quantify the cost-risk trade-off of different borrowing strategies. Some debt managers have made publicly available their research on these models. For example, the current simulation model employed by the central bank of Denmark is discussed in *Danmarks Nationalbank* (2005). The model uses a two-factor Cox, Ingersoll and Ross (CIR) (1985) yield curve model for the simulation of the interest rates and then compares debt strategies over a 10-year horizon, taking into account the Government's financing requirement forecasts.

Bergström and Holmlund (2000) and Bergström, Holmlund and Lindberg (2002) describe the simulation model constructed by the Swedish National Debt Office. The model uses a macroeconomic model that is similar in spirit to the one presented here, but in addition, it contains an external sector as Sweden issues foreign

[2] *Debt and Reserves Management Report 2006-07*, **HM Treasury 2006**.

currency denominated debt. The yield curve used in the model is a linear interpolation between a short and a long term yield.

Bolder (2002, 2003) describes the simulation model developed by the Bank of Canada. This model is a combined macro-yield curve model using a Markov-switching approach for the real GDP growth rate, a CIR yield curve model for the simulation of the interest rates and an equation that specifies the Government's financing requirement.

A useful overview of stochastic debt strategy simulation modelling in OECD countries can be found in Risbjerg and Holmlund (2005). The simulation model presented in this paper can be viewed as another contribution to this small extant literature on debt strategy stochastic simulation modelling.

The structure of the paper is as follows. The next section sets out the main features of the simulation model. Next, some illustrative results are presented. The paper then concludes with some final remarks.

2 Stochastic simulation model

Several factors influence the cost of servicing the government debt: the size and composition of the debt portfolio, the state of the real economy, the term structure of interest rates, inflation and the financing requirement of the Government. The simulation model captures in a highly stylised fashion how these factors interact to determine the debt cost of the Government.

The simulation framework consists of three main building blocks: (i) a *macroeconomic model* in which the output gap, the Government's primary net financing requirement, RPI and CPI inflation and the short interest rate are modelled as separate but inter-related equations; (ii) *yield curve models* which provide the specification for both the nominal and real term structure of interest rates; and (iii) the *debt strategy simulation* component, which is used to determine how, under a given debt strategy, the Government meets its total financing requirement (net central government cash requirement plus the refinancing of maturing debt). This latter component of the simulation model is also used to compute the cost and risk measures associated with the respective debt strategies, given the simulated path for the economy, the Government's financing requirement, interest rates and inflation.

2.1 Macroeconomic model

The macroeconomic part of the simulation model is made up of a small, trend-deviating model, which is in the spirit of the new-Keynesian models that have been developed for the analysis of monetary policy. The model is comprised of five equations that describe the behaviour of the output gap, the Government's primary net financing requirement, the CPI and RPI inflation, and the short interest rate. For simplicity, the current specification of the model is purely backward looking.

2.1.1 Economic cycle, output gap, and net primary financing requirement

The economic cycle is modelled as a simple two-state Markov switching regime (Hamilton (1989) and Goodwin (1993)) for the output gap – the deviation of actual output from potential output. Hence the typical behaviour of the economy is expressed as a stylised process with cyclical swings between above trend output and below trend output. Potential (trend) growth is assumed to be 2.5 percent per annum.

Specifically, the output gap is expressed as a function of the lagged real short interest rate and the lagged output gap as shown in equation (1) below:

$$y_t = \alpha_t + \rho y_{t-1} - \beta (r_{t-1}(0) - cpi_{t-1}) + \varepsilon_{y,t}, \ \varepsilon_{y,t} \sim N(0, \sigma_y^2) \qquad (1)$$

where t indicates time and t =1, 2,….,T, $y_t \equiv \frac{Y_t^r}{Y_t^T} - 1$= the output gap ($Y_t^r$ is actual real output or real GDP and Y_t^T is trend output or trend real GDP), α_t is a Markov switching intercept with two states or regimes α_1 and α_2 and the transitional matrix $A = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$, $r_t(0)$ = the short interest rate, cpi_t = the CPI inflation and ρ and β are parameters that measure the degree to which the output gap is affected by its previous value and the real short interest rate in the previous period, and $\varepsilon_{y,t}$ is an independent and normally distributed error term with zero mean and constant variance, σ_y^2 .[3] The transitional probabilities for the two regimes – above trend output and below trend output - imply that they both have identical average durations of about 10 quarters or 2.5 years.

Modelling the economic cycle is important for the analysis of debt strategies because it impacts on the other variables in the economy. For example, the term structure of interest rates tends to vary systematically over the economic cycle. Therefore the *unit costs* associated with the issuance strategies selected to meet the Government's financing requirement vary systematically over the economic cycle.

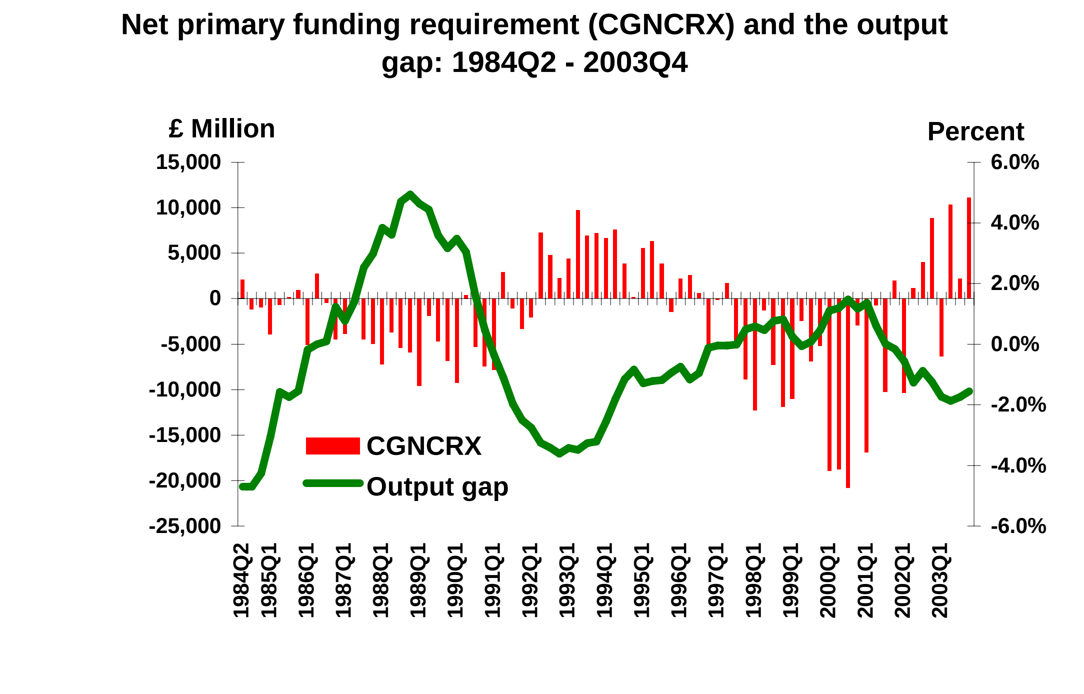
Also, as Figure 1 shows the Government's primary net financing requirement varies with the economic cycle. During periods of above trend output the primary net financing requirement tends to be in surplus (or show smaller than average deficits) because the Government's finances tend to be healthier as a consequence of higher tax revenues and lower expenditure. Conversely, in periods of below trend output the primary net financing requirement tends to be in deficit (or display smaller than average surpluses) because the Government's finances tend to be less healthy due to lower tax revenues and higher expenditure. Hence the *quantity* of the Government's financing requirement varies over the economic cycle. In modelling the Government's primary net financing requirement the influence of the economic cycle is therefore incorporated. As the simulation model is intended to reflect the salient features of the UK economy it includes the Government's two fiscal rules – the golden rule and the sustainable investment rule - in the modelling of the primary net financing requirement[4]. For this reason the Government's primary net financing requirement (as a share of GDP) is modelled as a function of the lagged output gap, lagged primary net financing requirement and the deviation of the lagged debt/GDP ratio from an assumed "long-run" average debt/GDP ratio:

---

[3] Strictly speaking, the variables should be superscripted by an indicator *i*, *i* = 1, 2,…., R to denote the replications in the simulation. However, the variables that are changing over the simulations are also those that are changing over time, which are already denoted by the subscript *t*. Hence, the indicator *i* is dropped to avoid overly cluttering the notation.

[4] The golden rule states that over the economic cycle the Government will only borrow to invest and not to fund current spending; and the sustainable investment rule states that the public sector net debt as a proportion of GDP will be held over the economic cycle at a stable and prudent level. Other things being equal, net debt will be maintained below 40 percent of GDP over the economic cycle (see ***Pre-Budget Report 2005***, **HM Treasury 2005**).

Figure 1



Source: HM Treasury.
Note: The net primary funding requirement (CGNCRX) is the central government net cash requirement (CGNCR) excluding interest payments.

$$f_t = \mu + \nu f_{t-1} - \pi y_{t-1} - \theta(d_{t-1} - d^*) + \varepsilon_{f,t}, \quad \varepsilon_{f,t} \sim N(0, \sigma_f^2) \quad (2)$$

where μ is a constant, $f_t \equiv \dfrac{F_t}{Y_t^n}$ = the primary net financing requirement, F_t as a share of nominal GDP (Y_t^n), $d_{t-1} \equiv \dfrac{D_{t-1}}{Y_t^n}$ = the debt/nominal GDP ratio in the previous period, $d^* \equiv \dfrac{D^*}{Y^{n*}}$ = the long-run average debt/nominal GDP ratio, which is set equal to 0.33 (33 percent), $\varepsilon_{f,t}$ is an error term and ν, π, and θ are the parameters that indicate respectively the extent to which the primary net financing requirement is influenced by its previous value, the output gap in the preceding period and the extent to which the Government has to change its fiscal policy in order to ensure that the debt/GDP ratio does not deviate too far from the long-run average ratio.

In should be pointed out that the above specification for the primary net financing requirement provides a stylised representation of both the golden and sustainable investment rules. There is no explicit current deficit in the model and therefore the golden rule is approximated by the assumption that over the "long-run" (and not necessarily over every economic cycle) the average primary net financing requirement must be in surplus. This means that the expected long-run value of f_t is negative (shows a surplus) in the primary net financing requirement equation (that is, $E(f_t) = \dfrac{\mu}{\nu - 1} < 0$).

The sustainable investment rule is represented in the model by the restriction that the average long-run debt/GDP ratio is equal to the starting debt to GDP ratio. The model maintains the long-run debt ratio, on average, in a symmetrical manner, expressed by the primary net financing requirement adjusting accordingly (through the term $-\theta(d_{t-1} - d^*)$) when the actual debt/GDP ratio diverges from the initial debt/GDP ratio. When the actual debt/GDP ratio exceeds the initial debt/GDP ratio, the Government tightens its fiscal stance and generates a larger primary net financing requirement surplus; conversely when the actual debt/GDP ratio falls below the initial debt/GDP ratio, the Government relaxes its fiscal stance and generates a larger primary net financing requirement deficit.

In contrast, the sustainable investment rule as actually set by the Government is asymmetrical with only an upper limit set for the public sector net debt to GDP ratio[5] over the economic cycle. Moreover, the simulation model identifies economic states and thus does keep track of the economic cycles through time. The fiscal rules as represented in the net primary financing requirement equation are only observed as long-run properties of the model. To ensure that the fiscal rules are met over the economic cycle would require some form of dynamic programming and that implies a much more complex model framework than the current model.

2.1.2 CPI inflation, RPI inflation and the short interest rate

The simulation model is to be used to examine borrowing strategies that reflect the choice of debt instruments currently available to the Government. The Government issues both nominal gilts and inflation-linked gilts. In order to capture the inflation compensation on both the coupon payment and the outstanding principal payable on inflation-linked bonds, we need to specify the price process.

Both CPI and RPI inflation are modelled. In order to reflect the current monetary policy regime CPI inflation is targeted by the central bank. We assume that the CPI inflation target is fully credible and the expected CPI inflation is set to be consistent with the current Bank of England target of 2%. CPI inflation is modelled as a Phillips curve and it is expressed specifically as a linear function of the lagged output gap and lagged CPI inflation:

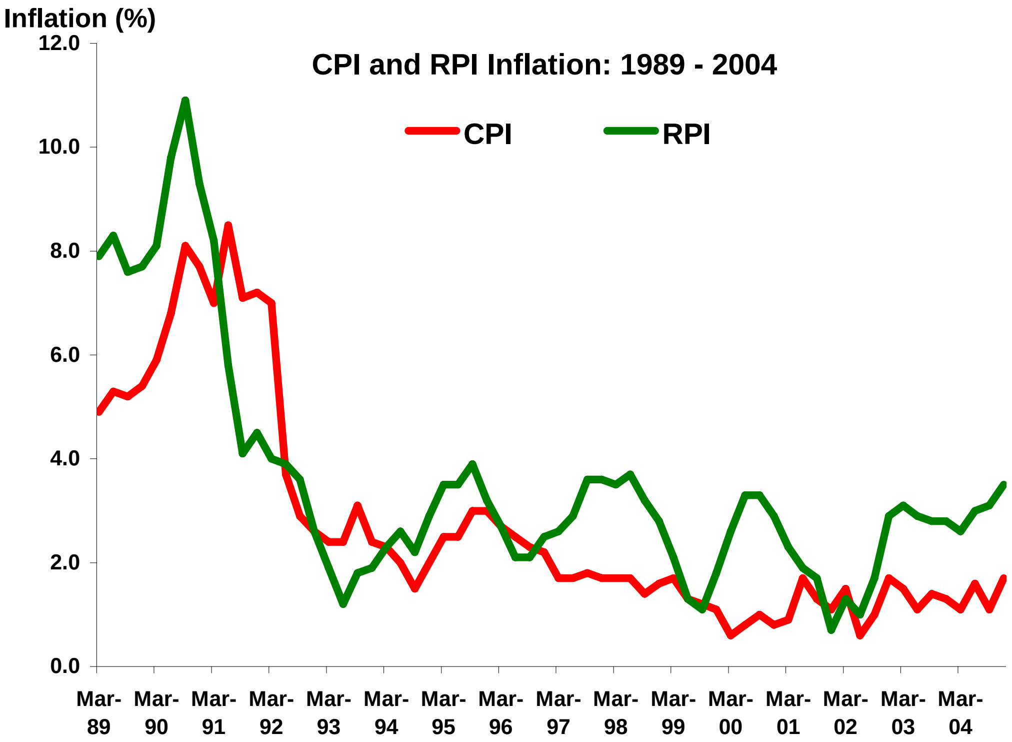
$$cpi_t = \varsigma(1-\xi) + \xi\, cpi_{t-1} + \psi\, y_{t-1} + \varepsilon_{cpi,t}, \quad \varepsilon_{cpi,t} \sim N(0, \sigma^2_{cpi}) \qquad (3)$$

where ς is the inflation target of the central bank, ξ and ψ are respectively parameters that measure the strength with which CPI inflation is influenced by its previous value and the value of the output gap in the preceding period and $\varepsilon_{cpi,t}$ is an error term.

As inflation-linked bonds are tied to the RPI index, it is necessary to model RPI inflation in order to calculate the inflation compensation on these bonds. It is reasonable to assume that there are systematic differences between CPI inflation and RPI inflation over the economic cycle, as can be seen from Figure 2. One reason for this is that changes in the short interest rate made by the central bank in its attempt to stabilise CPI inflation at its target level tend to have an impact on RPI inflation (see Figure 3).

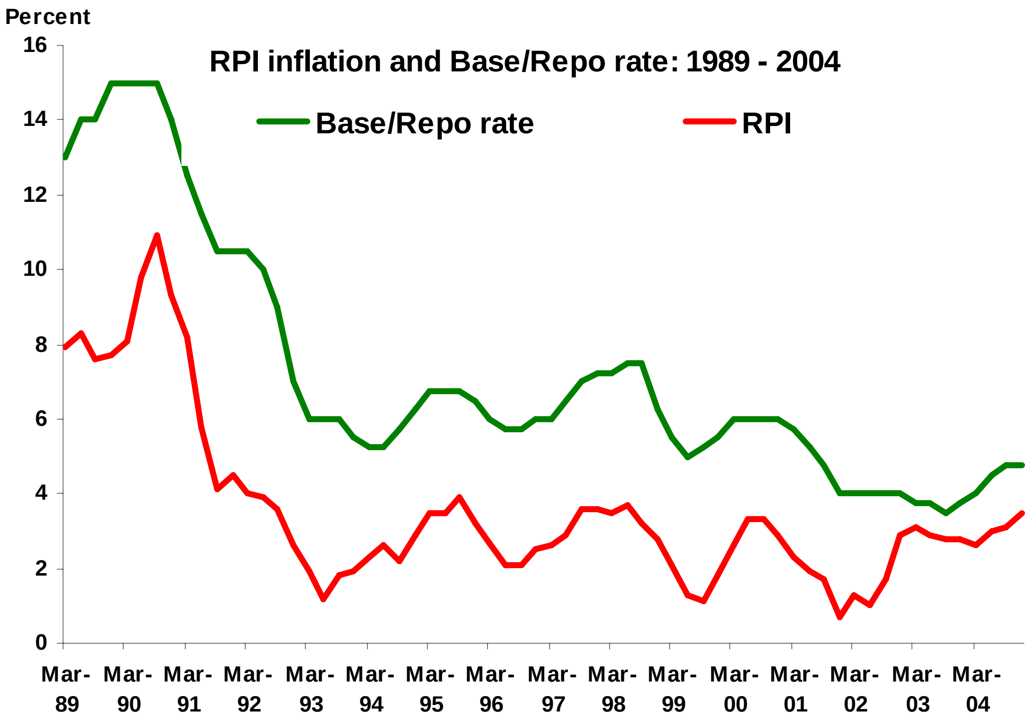
[5] The sustainable investment rule is defined in terms of the public sector net debt to GDP ratio. In contrast, the model uses the gross debt ratio.

Figure 2



Source: ONS

Figure 3



Source: ONS and Bank of England

One channel through which this effect occurs is via the impact of adjustments in the central bank's policy rate on mortgage interest rates and consequently mortgage interest payments, which are included in the RPI index. RPI inflation is therefore modelled as a function of contemporaneous CPI inflation and the short interest rate:

$$rpi_t = \kappa + cpi_t + \iota r_t(0) + \varepsilon_{rpi,t}, \ \ \varepsilon_{rpi,t} \sim N(0, \sigma^2_{rpi}) \qquad (4)$$

where κ is a constant, ι indicates the extent to which the short interest rate affects RPI inflation and $\varepsilon_{rpi,t}$ is an error term.

To complete the macroeconomic part of the simulation model, we specify the evolution of the short interest rate. The short interest rate, from a macroeconomic perspective, is the policy rate under the direct control of the central bank, which varies the rate in pursuit of its objective of stabilising CPI inflation at the 2% target. Another important reason for modelling the short interest rate is that it is an important building block for the interest rates at other maturities, which are risk-adjusted averages of expected future short interest rates. Hence, changes in the short interest rate influence the variations in the interest rates at other maturities. The short interest rate is modelled as a simple Taylor rule, and it is expressed as a function of the lagged output gap and the lagged CPI inflation:

$$r_t(0) = \phi + \omega\, cpi_{t-1} + \chi\, y_{t-1} + \varepsilon_{r(0),t}, \ \ \varepsilon_{r(0),t} \sim N(0, \sigma^2_{r(0)}) \qquad (5)$$

where ϕ is a constant, ω and χ respectively show the degree to which the previous period's value of CPI inflation and the lagged value of the output gap cause the central bank to vary the short interest rate and $\varepsilon_{r(0),t}$ is an error term.

The values for the parameters of the macroeconomic model are derived from a combination of estimation, theory, and calibration. The estimation uses quarterly data for the UK economy over the period 1992 - 2004. Theoretical restrictions are imposed on the parameters of the model so that, for example, the long-run average CPI inflation is constrained to be equal to the inflation target under the assumption of a credible monetary policy regime and the output gap averages to zero. The parameterisation of the macroeconomic part of the simulation model captures therefore in a highly stylised fashion some of the main features of the UK economy over the recent past. Table 1 summarises the parameterisation of the macroeconomic model.

2.2. Yield curve models

As the Government funds its total borrowing requirement by issuing bonds it is required that the interest rates at which it issues these bonds be computed. Further, since the Government can choose between conventional fixed rate bonds and inflation-linked bonds, it is necessary to model the interest rates for each type of bond. In order to price the coupons of the bonds issued, the simulation model requires a yield curve for conventional bonds and one for index-linked bonds. The yield curve for conventional bonds is based on the yield curve function introduced by Nelson and Siegel (NS) (1987), and it is specified so as to capture the influence of macroeconomic developments on the evolution of the term structure of interest rates.

The real yield curve is derived from the nominal yield curve under the assumption of fixed inflation expectations. This assumption is fairly plausible because, in the

Table 1: The parameterised equations of the macroeconomic model

$$y_t = \alpha_t + 0.1y_{t-1} - 0.05(r_{t-1}(0) - cpi_{t-1}) + \varepsilon_{y,t},$$
$$\varepsilon_{y,t} \sim N(0, 0.0015^2)$$

$$f_t = -0.000000135 + 0.55f_{t-1} - 0.5y_{t-1} - 0.02(d_{t-1} - d^*) + \varepsilon_{f,t},$$
$$\varepsilon_{f,t} \sim N(0, 0.0008^2)$$

$$cpi_t = 0.00496(1-0.3) + 0.3cpi_{t-1} + 0.2y_{t-1} + \varepsilon_{cpi,t},$$
$$\varepsilon_{cpi,t} \sim N(0, 0.0001^2)$$

$$rpi_t = -0.003 + cpi_t + 0.5r_t(0) + \varepsilon_{crpi,t},$$
$$\varepsilon_{rpi,t} \sim N(0, 0.0001^2)$$

$$r_t(0) = 0.003 + 1.5cpi_{t-1} + 0.5y_{t-1} + \varepsilon_{r(0),t},$$
$$\varepsilon_{r(0),t} \sim N(0, 0.002^2)$$

where α_t is a Markov switching intercept with two states α_1 = -0.0025 and α_2 = 0.0029952 and the transition matrix $A = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$. The variances of the error terms in the respective equations, excepting the primary net financing requirement equation, are set such that the variances of the variables in the model are similar to their empirical variances. In addition, the model parameters are set so that the model corresponds to quarterly data. This means, for example, that the CPI inflation target of 2% translates into a model parameterisation of $(1+0.02)^{0.25}-1 \approx 0.005$.

---

simulation exercises, index-linked bonds are only issued at 10-year and 30-year maturities and expectations for inflation 10 years and beyond into the future within the same credible monetary policy framework are likely to be relatively well anchored, as we have assumed.[6]

The nominal and real yield curves are modelled from a combination of theoretical considerations and empirical evidence. Importantly, the yield curves are modelled so that on average they are inverted at the long end, consistent with the average shape of the UK yield curves over the period 1998 - 2004.

[6] Strictly speaking, the assumed credibility of the monetary regime implies that expectations of future CPI inflation will be well anchored at the central bank's CPI inflation target. For our purposes, we require expectations of RPI inflation to be well anchored also. This we achieve by further assuming that there is a stable relationship between CPI and RPI inflation and therefore well anchored long run expectations of CPI inflation mean also stable long run expectations of RPI inflation.

2.2.1 Nominal yield curve

The nominal yield curve is based on the NS yield curve function. Diebold and Li (2006) have recently reinterpreted the NS yield curve function as a three-factor yield curve model and it is this latter re-interpretation that we adopt in the nominal yield curve specification.

The use of factor models in yield curve modelling is quite a common practice. There are several reasons for researchers adopting this approach. As Diebold, Piazzesi and Rudebusch (2005) point out, one important reason is that factor models provide a convenient way of summarising the voluminous yield information contained in the large number of bonds that are traded at any point in time. Another reason is that factor models, in allowing the compression of information, is consistent with the "parsimony principle" which broadly implies that imposing restrictions on models, and thereby constraining them in some way, can be useful for producing good forecasting models.

The NS yield curve has the following functional form:

$$
\begin{aligned}
r_t(\tau) &= l_t + s_t\left(\frac{1-\exp(-\tau/\lambda)}{\tau/\lambda}\right) \\
&+ c_t\left(\frac{1-\exp(-\tau/\lambda)}{\tau/\lambda} - \exp(-\tau/\lambda)\right) + \varepsilon_t(\tau), \quad \varepsilon_t(\tau) \sim N\left(0, \sigma(\tau)^2\right)
\end{aligned}
\tag{6}
$$

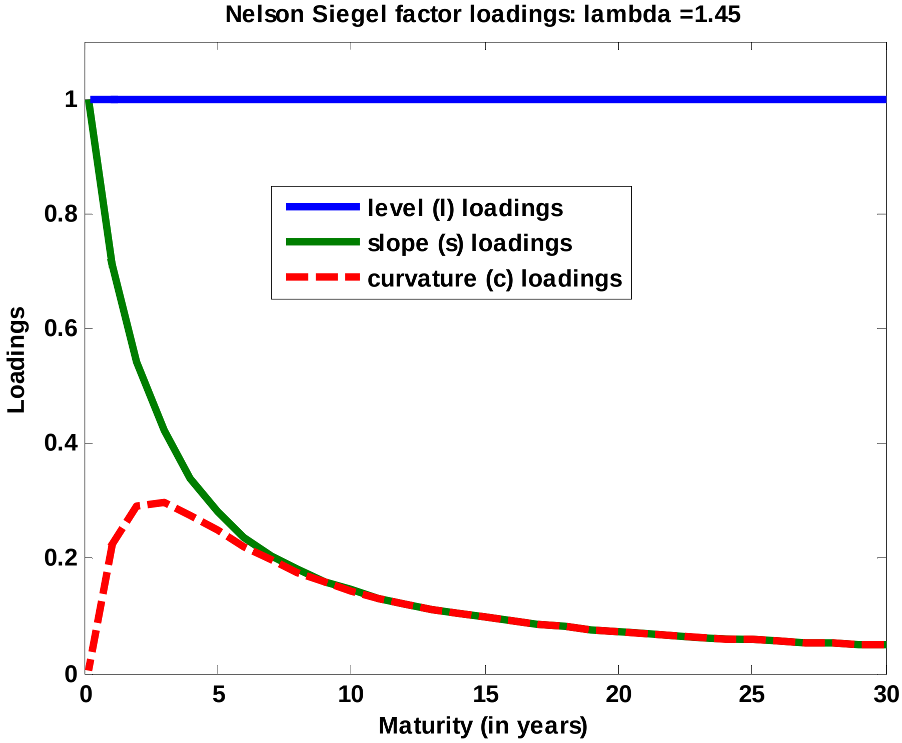
where $r_t(\tau)$ denotes the yield to maturity τ at time *t* and l_t, s_t, c_t and λ[7] are parameters that determine the shape of the yield curve and $\varepsilon_t(\tau)$ is an error term.

Following Diebold and Li (2006), equation (6) shows that the NS yield curve is a linear combination of the three functions or factor loadings – 1, $\left(\frac{1-\exp(-\tau/\lambda)}{\tau/\lambda}\right)$ and $\left(\frac{1-\exp(-\tau/\lambda)}{\tau/\lambda} - \exp(-\tau/\lambda)\right)$ - with their corresponding latent or unobserved dynamic parameters or factors l_t, s_t and c_t. The three latent dynamic factors l_t, s_t and c_t are respectively considered level, slope and curvature factors as it can be shown, given their factor loadings, that they influence these three latent elements of the yield curve as Figure 4 shows. This re-interpretation of the NS model is indeed quite insightful because traditional factor analysis has, since the work of Litterman and Scheinkman (1991), shown that much of the variation in bond yields can be explained by the first three principal components, which have been interpreted as level, slope and curvature factors.

For our purposes, we are interested in having a yield curve model that can be used to simulate interest rates at different maturities over time and that can also link the evolution of these interest rates to the developments in the economy, so that there is

[7] Strictly speaking λ is also a time varying parameter; but as it is common practice to fix it, we have dropped the time subscript *t*. We have set λ = 5.8 in the simulations.

Figure 4



Note: The figure plots the factor loadings in the NS model given in equation (6). The three factors are l_t, s_t and c_t and their corresponding loadings are 1, $\left(\frac{1-\exp(-\tau/\lambda)}{\tau/\lambda}\right)$ and $\left(\frac{1-\exp(-\tau/\lambda)}{\tau/\lambda}-\exp(-\tau/\lambda)\right)$. λ = 1.45. Maturity is shown in years rather than in quarters as in the simulation in order to be consistent with the other figures showing yields in the paper.

an economic explanation for the behaviour of the yield curve over time rather than a purely statistical one. As our nominal yield curve specification is a mixture of estimation and calibration based on theoretical considerations our approach to capturing the yield curve dynamics is different from both the two-step estimation procedure of Diebold and Li (2006) and the state space modelling approach of Diebold, Rudebusch and Aruoba (2006), in which they also incorporate macroeconomic variables in the state space system.

In our NS model specification, we link the evolution of the three dynamic latent factors directly to the evolution of the short interest rate ($r_t(0)$), the CPI inflation (cpi_t) and the output gap (y_t).

Specifically, the three latent factors l_t, s_t and c_t are assumed to be determined by the short interest rate ($r_t(0)$), the CPI inflation (cpi_t) and the output gap (y_t):

$$\Gamma \mathbf{f}_t = \boldsymbol{\mu}_f + \mathbf{A}\mathbf{m}_t + \boldsymbol{\eta}_t \qquad (7)$$

where $\mathbf{f}_t = \{l_t, s_t, c_t\}$, $\mathbf{m}_t = \{y_t, r_t(0), cpi_t\}$, $\boldsymbol{\mu}_f$ is a (3x1) vector, Γ and $\mathbf{A}$ are (3x3) matrices of parameters and $\boldsymbol{\eta}_t$ is a (3x1) vector of error terms.

The parameters are chosen by the following combination of theoretical considerations and empirical evidence. Apart from affecting all yields equally l_t is also a long term factor as its loading is unity, a constant and it does not decay to zero in the limit. Hence $r_t(\tau)^{\tau\to\infty} = l_t$. We impose the restriction that l_t is equal to the expected (long-run average) short interest rate, so that $l_t = E(r_t(0))$.

The parameter s_t, as was indicated previously, gives the (negative) slope of the yield curve as $r_t(\tau)^{\tau\to 0} = l_t + s_t$, and $r_t(\infty) - r_t(0) = -s_t$. At maturity τ = 0 the yield curve should be equal to the short interest rate ($r_t(0)$) determined by the Taylor rule. Therefore it can be assumed that $l_t + s_t = r_t(0)$. The relationship for the curvature factor c_t does not have the same theoretical solution as the other two latent factors and it is therefore estimated empirically.

The following two-step procedure is used to estimate the parameters for the curvature factor c_t. We assume the following specification for the three factors:

$$\begin{bmatrix} l_t \\ s_c \\ c_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_{t-1} \\ s_{t-1} \\ c_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_{lt} \\ \eta_{st} \\ \eta_{ct} \end{bmatrix} \quad (8)$$

where the error terms are *iid* normal with a given variance σ_{jt}^2 $j = l, s, c$. The factors are estimated in a state space model with equation (8) as the measurement equation and equation (6) as the transition equation using the EM-algorithm in conjunction with the Kalman filter as described by Harvey (1989).

The curvature factor obtained from the Kalman filter is then regressed on an intercept, the output gap, the short interest rate and CPI inflation. The estimated parameters are 0.03 for the intercept, 0.7 (2.9) for the output gap, -0.6 (2.7) for the short interest rate and -2.2 (3.6) for the CPI inflation, where the terms in brackets are the *t*-statistics.

Combining the theoretical considerations for the level and slope factors and the estimated relationship for the curvature factor gives us the following reduced form equation for the nominal yield curve parameters in the simulation model:

$$\begin{bmatrix} l_t \\ s_c \\ c_t \end{bmatrix} = \begin{bmatrix} E(r_t(0)) \\ -E(r_t(0)) \\ 0.03 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0.7 & -0.6 & -2.2 \end{bmatrix} \begin{bmatrix} y_t \\ r_t(0) \\ cpi_t \end{bmatrix} + \begin{bmatrix} \eta_{lt} \\ \eta_{st}^* \\ \eta_{ct} \end{bmatrix} \quad (9)$$

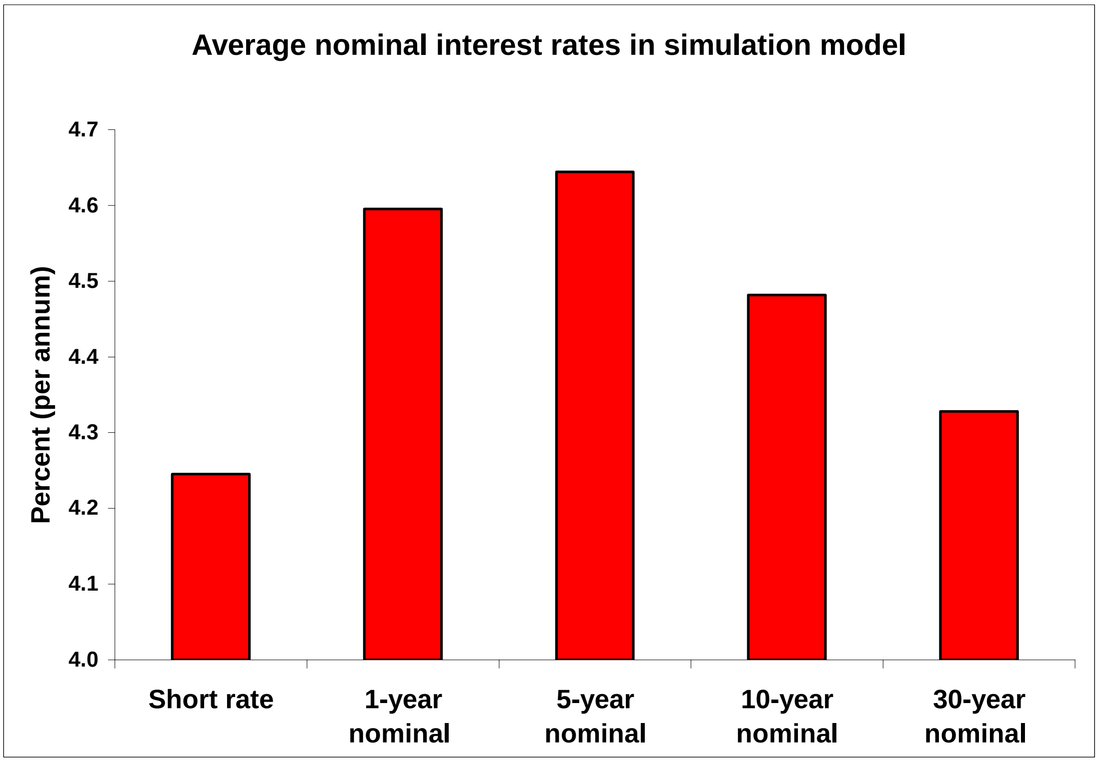
where $\eta_{st}^* = \eta_{st} - \eta_{lt}$.

Table 2 and Figure 5 show that the averages of the simulated nominal yields and their volatility are similar to those of the nominal yield curve over the period 1998-2004.

Table 2: Actual (1998Q1 – 04Q4) and simulated interest rates (in %)

Interest Rates	Actual		Simulated	
	Mean	Std. Deviation	Mean	Std. Deviation
Short rate	5.0803	1.1786	4.2453	1.1581
1-year	4.6937	0.8560	4.5951	0.6928
5-year	4.9038	0.6572	4.6444	0.6180
10-year	4.8475	0.4280	4.4776	0.3855
30-year	4.5373	0.2144	4.3280	0.1992

Figure 5



2.2.2. Real yield curve

The real yield curve is derived from the nominal yield curve under the assumption of fixed inflation expectations, which, as was indicated earlier, in this case is not unrealistic as index-linked bonds are only issued at medium and long maturities, and expectations for inflation 10 years into the future within the same credible monetary policy framework are likely to be relatively constant. The real yield curve is specified as

$$r_t^r(\tau) = r_t(\tau) - rpi^e + l_t^r + s_t^r\left(\frac{1-\exp(-\tau/\lambda)}{\tau/\lambda}\right) + c_t^r\left(\frac{1-\exp(-\tau/\lambda)}{\tau/\lambda} - \exp(-\tau/\lambda)\right) + \varepsilon_t^r(\tau), \tag{10}$$

where rpi^e is the long run expected RPI inflation,

$$rpi^e = E(rpi_t) = \kappa + \iota\phi + (1+\iota\omega)\varsigma \tag{11}$$

and l_t^r , s_t^r and c_t^r are respectively the corresponding level, slope and curvature factors for the real yield curve and they are determined by

$$\begin{bmatrix} l_t^r \\ s_t^r \\ c_t^r \end{bmatrix} = \begin{bmatrix} \eta_{lt}^r \\ \eta_{st}^r \\ \eta_{ct}^r \end{bmatrix} \tag{12}$$

where the error terms are *iid* normal with a given variance σ_{it}^2 *i* = l^r , s^r and c^r .

In the simulations, it is therefore assumed that the nominal and real yield curves only differ on average by the long run expected RPI inflation rpi^e .

The current specification of the yield curve models does not allow for any potential influence on the yields of the respective bonds that arises from changes to their relative supplies. The implication is that, in the model, issuance strategies can be composed of any combination of bonds without having any consequence for the evolution of yields over time. This is certainly a limitation of the model as it is appears that changes in the relative supply of bonds will tend to influence yields.

2.3 Debt strategy simulation

The debt strategy component of the model controls how the Government borrows to meet its total financing requirement in any given period. The total financing requirement for any given period is equal to the sum of the modelled primary net financing requirement, interest payments and redemptions. Interest payments and redemptions are obtained directly from the information on outstanding debt in a given portfolio.

For simplicity, issuance is always composed of new bonds. There is therefore no re-opening of existing bonds in the debt strategy simulation. All bonds are issued at par and thus the yield at issuance is always equal to the coupon. We set coupon payments consistent with the frequency of the simulation model so that they are paid quarterly rather than semi-annually, as is the current convention. This also means that the uplift on interest payments on inflation-linked bonds is on a quarterly basis and lagged one quarter.

The starting debt-to-GDP ratio is set at 0.33 (33% of GDP) and for each borrowing strategy this is converted into an initial debt portfolio that is composed of the bonds in proportion that matches identically the borrowing strategy.

The initial values for the macroeconomic variables in the simulations are their respective long-run expected values:

$y_{t=0} = E(y_t) = 0$;

$f_{t=0} = E(f_t) = \dfrac{\mu}{\nu - 1}$;

$cpi_{t=0} = E(cpi_t) = \varsigma$;

$rpi_{t=0} = E(rpi_t) = \kappa + \iota\phi + (1 + \iota\omega)\varsigma$; and

$r_{t=0}(0) = E(r_t(0)) = \phi + \omega\varsigma$.

The starting nominal and real yield curves take the initial values of relevant macroeconomic variables as their inputs.

The model is simulated over a period of 125 years (500 quarters) with 2000 replications. We use both the observations in the 500th quarter and the observations over the last 100 quarters of the simulation interval to compute the statistics that we use to compare issuance strategies.

Figure 6 provides a graphical overview of the simulation model.

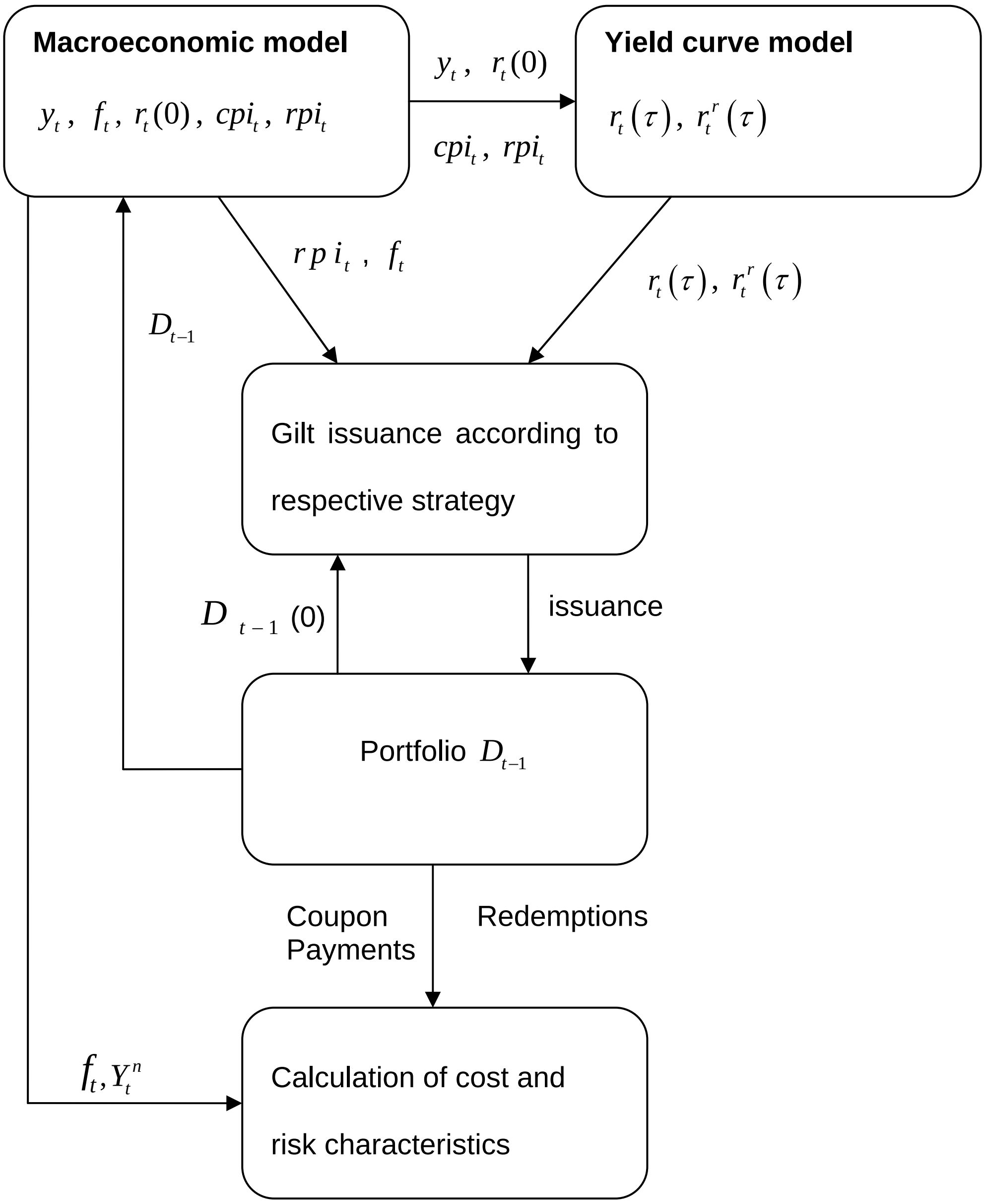
2.4 Cost and risk measures

It is necessary to define what is meant by cost and risk in order to be able to compare debt strategies on the basis of their cost-risk trade-off. The cost of the debt in any given period is defined in cash flow terms and is computed as the sum of all nominal coupon payments (interest payments on nominal bonds plus inflation compensated interest payments on inflation-linked bonds) plus the realised inflation compensation effects on maturing inflation-linked bonds:

$$CS_t = CI_t^N + CI_t^{IL} + CP_t^{IL} \tag{13}$$

where CS_t = cost of the debt in period *t*, CI_t^N = coupon payments on nominal bonds, CI_t^{IL} = inflation compensated coupon payments on inflation-linked bonds and CP_t^{IL} = the inflation compensation paid on the principal of maturing inflation-linked bonds.

Figure 6: Graphical representation of the simulation model



Note: y_t: output gap; $r_t(\tau)$ and $r_t^r(\tau)$: nominal and real interest rates respectively with maturity λ; cpi_t and rpi_t: CPI and RPI inflation respectively; f_t: primary net financing requirement; D_{t-1}: debt stock; D_{t-1}(0): maturing debt stock; and Y_t^n: nominal GDP.

We measure debt cost as a proportion of nominal GDP and equation (13) becomes:

$$cs_t = ci_t^N + ci_t^{IL} + cp_t^{IL} \tag{14}$$

where $cs_t = \frac{CS_t}{GDP_t}$, $ci_t^N = \frac{CI_t^N}{GDP_t}$, $ci_t^{IL} = \frac{CI_t^{IL}}{GDP_t}$ and $cp_t^{IL} = \frac{CP_t^{IL}}{GDP_t}$.

There are advantages to using this *debt cost ratio* rather than the nominal cost of the debt. First, the debt cost ratio gives a better picture of the Government's financial situation in that it provides a clearer indication of the debt cost burden to the Government than does the nominal cost of debt on its own. Second, the debt cost ratio is consistent with the Government's fiscal rules, in particular the sustainable investment rule, which relates the public sector net debt to nominal GDP. Third, the debt cost ratio provides a rudimentary way of capturing an asset and liability management (ALM) approach to government debt management in that the cost of the debt is related to the source from which the Government secures its tax revenues, which are its principal asset.

The risk measures we use in the model capture the concept of *financing risk*, the uncertainty in the financing or cash flow cost related to a given borrowing strategy. The financing risk associated with a given debt strategy is evaluated by two statistics. The first statistic is the standard deviation of the debt cost ratio, which measures the volatility of the debt cost ratio:

$$\sigma_{cs,t} = \left(\frac{1}{R-1} \sum_{i=1}^{R} \left(cs_{it} - \mu_t^{cs} \right)^2 \right)^{\frac{1}{2}} \qquad (15)$$

where $\sigma_{cs,t}$ = standard deviation of the debt cost ratio in period *t*, cs_{it} = debt cost ratio in period *t* for the *i*-th replication in the simulation and $\mu_t^{cs} = \frac{1}{R}\sum_{i=1}^{R} cs_{it}$ = mean debt cost ratio in period *t*.

The second statistic is the upper 95th percentile of the debt cost ratio distribution that gives the largest debt cost ratio, such that it is exceeded by five percent of the debt cost ratio realisations. This latter statistic is in the spirit of the commonly used Value-at-Risk (VaR) approach used in finance and risk management and will accordingly be referred to as the debt cost ratio-at-risk. The debt cost ratio-at-risk is a useful risk measure especially when the Government is concerned about avoiding extremely high debt cost ratios. In contrast, the standard deviation measures risk symmetrically, in that it relates to deviations from the mean debt cost ratio.

Although the model directly measures financing risk, it is clear that this risk is closely related to the wider issue of *budget or fiscal risk*, the uncertainty in the budget position associated with the volatility in debt cost emanating from a given borrowing strategy. This is because the debt cost is one of the items of government expenditure and therefore variations in the debt service cost directly impact on the volatility in the Government's financial position. However, another important consideration also is the way in which debt service costs co-vary with the primary net financing requirement.

In general, in order to minimise its budget risk, the Government would ideally like to have in its portfolio debt instruments with the following features: (a) debt instruments with low debt service cost variability; (b) debt instruments with debt service costs that co-vary negatively with the debt service costs of other debt instruments in the portfolio (and thus providing insurance against variations in the debt service costs of other debt instruments in the debt portfolio) and (c) debt instruments with debt service costs that co-vary positively with the primary net financing requirement surplus. All other things being equal, feature (c) would imply that a debt portfolio that typically has low costs when the Government finances are strained is deemed less risky overall than a portfolio to which the opposite applies.

3 Illustrative Results

The debt strategies that are compared are fixed issuance rules that are composed of varying shares of nominal and inflation-linked bonds. Consistent with current issuance practice, nominal bonds are issued with short, medium and long maturities, but the issuance of inflation-linked bonds is restricted to medium and long maturities only.

These fixed issuance strategies do not take account of several features of the debt management process which are important elements of the UK Government's debt management strategy. For example, issuance strategies in the model are not motivated by the need to build up benchmark bonds in order to secure a benchmark premium and thereby lower the long-run cost of funding for the Government.

For illustration, we firstly compare four issuance strategies with varying shares of short, medium and long maturities nominal bonds only. The purpose of this exercise is to highlight how changes to the maturity structure of conventional issuance strategies affect the cost-risk trade-off faced by the Government, under the assumed conditions of the simulation model. The composition of these four conventional issuance strategies is as follows:

- Strategy 1 is made up of 17.5 percent of 1-year nominal bonds, 17.5 percent of 5-year nominal bonds, 30 percent of 10-year nominal bonds and 35 percent of 30-year nominal bonds.
- Strategy 2 is composed of 35 percent of 5-year nominal bonds, 30 percent of 10-year nominal bonds and 35 percent of 30-year nominal bonds.
- Strategy 3 is composed of 50 percent of 10-year nominal bonds and 50 percent of 30-year nominal bonds.
- Strategy 4 comprises only 30-year nominal bonds.

Table 3 summarises the composition of the four issuance strategies.

Table 3: Composition of issuance strategies (in %)

Strategy	1-year nominal bond	5-year nominal bond	10-year nominal bond	30-year nominal bond
Strategy 1	17.5	17.5	30	35
Strategy 2		35	30	35
Strategy 3			50	50
Strategy 4				100

Next, we provide cost and risk comparisons for 4 issuance strategies with varying shares of short and medium maturity nominal bonds and 30-year inflation-linked bonds. The purpose of this exercise is to highlight how changes to the composition of issuance strategies affect the cost-risk trade-off faced by the Government, under the assumed conditions of the simulation model. The composition of the four mixed conventional and inflation-linked issuance strategies to be compared is as follows:

- Strategy 5 is made up of 17.5 percent of 1-year nominal bonds, 17.5 percent of 5-year nominal bonds, 30 percent of 10-year nominal bonds and 35 percent of 30-year inflation-linked bonds.
- Strategy 6 is composed of 35 percent of 5-year nominal bonds, 30 percent of 10-year nominal bonds and 35 percent of 30-year inflation-linked bonds.
- Strategy 7 is composed of 50 percent of 10-year nominal bonds and 50 percent of 30-year inflation-linked bonds.
- Strategy 8 comprises only 30-year inflation-linked bonds.

Table 4 summarises the composition of the four issuance strategies.

Table 4: Composition of issuance strategies (in %)

Strategy	1-year nominal bond	5-year nominal bond	10-year nominal bond	30-year inflation-linked bond
Strategy 5	17.5	17.5	30	35
Strategy 6		35	30	35
Strategy 7			50	50
Strategy 8				100

3.1 Results for nominal issuance strategies

Table 5 presents the statistics on the debt costs associated with the 4 nominal issuance strategies. It can be seen that as the share of 30-year bonds increases in the issuance strategies the average debt cost falls and that the issuance strategy with only 30-year nominal bonds has marginally the lowest debt cost. This result is unsurprising and it is largely a consequence of the fact that interest rates on long nominal bonds are lower than interest rates on short and medium nominal bonds as Figure 5 and Table 2 show.

One way of illustrating this influence of the shape of the yield curve on the average debt cost of the various issuance strategies is to examine what the average cost of £1 of financing requirement would be under each issuance strategy. Figure 7 compares the average interest rates for each of the four strategies and it is clearly evident that the issuance rule that is made up of only 30-year nominal bonds (strategy 4) has the lowest average interest rates.

Table 5: Summary of simulation results for nominal issuance strategies

Strategy	Strategy 1	Strategy 2	Strategy 3	Strategy 4
Debt cost/GDP at t=500				
Mean	1.4148	1.4300	1.4072	1.3912
Std deviation	0.1966	0.2034	0.1959	0.1860
95th percentile	1.7548	1.7636	1.7364	1.7176
Debt cost/GDP over the interval t=400 to t= 500				
Mean Mean	1.4155	1.4304	1.4125	1.3887
Mean Std deviation	0.1962	0.2035	0.1954	0.1897
Mean 95th percentile	1.7327	1.7696	1.7379	1.7097

Note: The respective statistics shown for each strategy are defined as follows:

Mean = $\mu_t^{cs} = \frac{1}{R}\sum_{i=1}^{R} cs_{it}$, i = 1,2,…,2000 and t = 500

Std deviation = $\sigma_{cs,t} = \left(\frac{1}{R-1}\sum_{i=1}^{R}\left(cs_{it} - \mu_t^{cs}\right)^2\right)^{\frac{1}{2}}$,

95th percentile = $cs_{t=500}^{0.95}$ = upper 95th percentile of the debt cost ratio at t = 500

Mean Mean = $\mu^{cs*} = \frac{1}{T}\sum_{t=400}^{T=500}\frac{1}{R}\sum_{i=1}^{R} cs_{it}$,

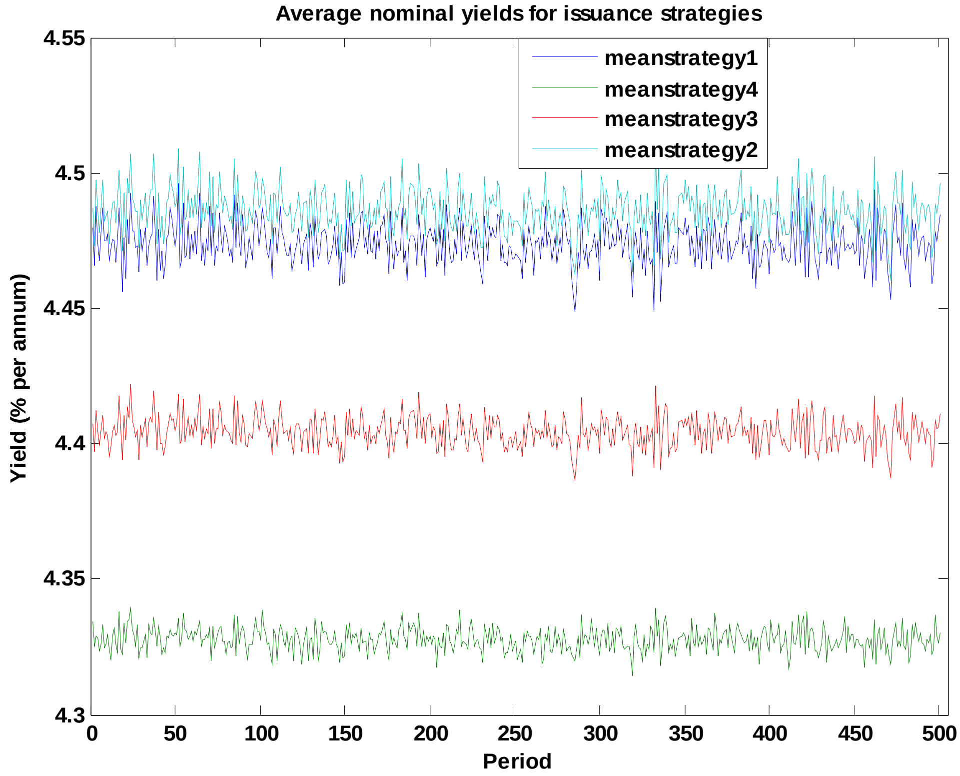
Mean Std deviation = $\sigma_{cs}^{*} = \frac{1}{T}\sum_{t=400}^{T=500}\left(\frac{1}{R-1}\sum_{i=1}^{R}\left(cs_{it} - \mu_t^{cs}\right)^2\right)^{\frac{1}{2}}$, and

Mean 95th percentile = $cs^{*} = \frac{1}{T}\sum_{t=400}^{T=500} cs_t^{0.95}$.

The figures are quarterly, annualised and expressed in percentage points.

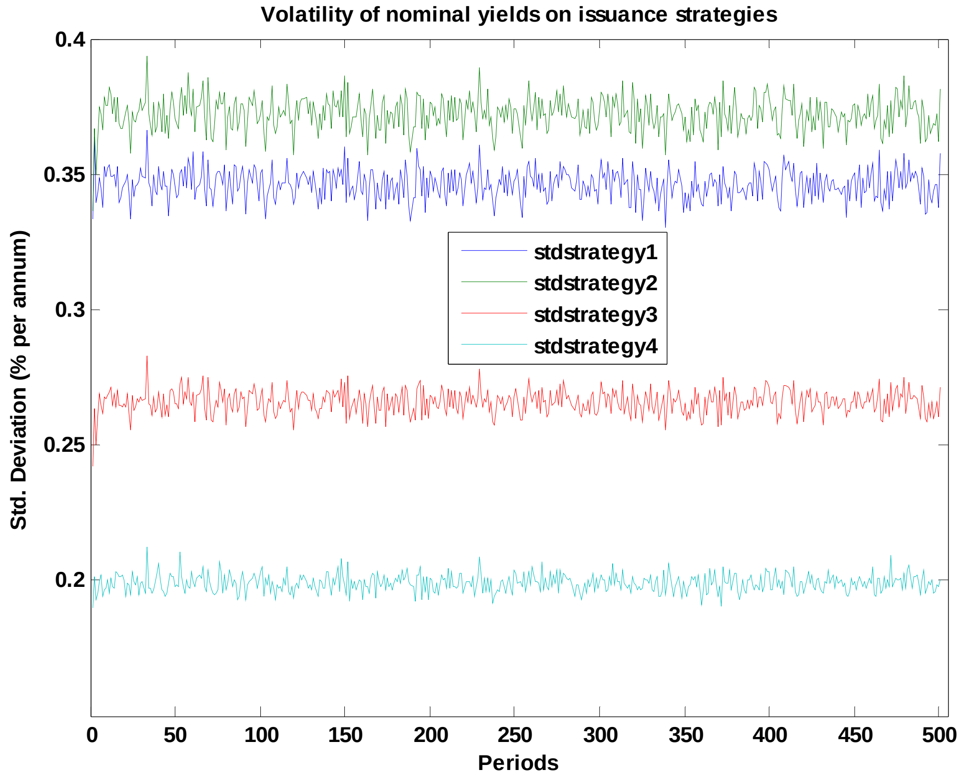
Regarding risk properties (standard deviation and the debt cost ratio-at-risk measure) the long conventional issuance strategy turns out to have both marginally lower standard deviation and cost-at-risk than the other three conventional strategies. Once again this result is unsurprising and can be partly attributed to the fact that the interest rate on 30-year nominal bonds has the lowest volatility of the nominal bonds and therefore an issuance strategy that is composed entirely of 30-year nominal bonds will show lower volatility than any other issuance strategies that contain mixtures of short and medium nominal bonds. This is shown in Figure 8, which compares the volatility of the interest rates for the four nominal issuance strategies.

Figure 7



.

Figure 8



It is worth emphasising that the foregoing results, as with results from any model, are the outcome of the crucial assumptions made in constructing the model. It is straightforward to show that modifying the assumptions about the term structure of interest rates would lead to different conclusions about the ranking of the issuance strategies. For example, if the yield curve is upward sloping (and there is a positive term spread) then longer dated bonds are more expensive than shorter maturity bonds and this would lead to different conclusions about the issuance strategy that would have the lowest cost.

To illustrate this point the four nominal issuance strategies are compared assuming that the nominal yield curve is upward sloping (all of the other model assumptions and parameterisation remain unchanged).[8] The average nominal yields and volatilities on the respective bonds for this new nominal yield curve are shown in Table 6 and Figure 9 respectively. Observe that the new nominal yield curve is also more volatile than in the previous example and that the volatility of the interest rates on the respective bonds generally fall with maturity up to the 10-year tenor, but the 30-year nominal yield is slightly more volatile than the 10-year yield.

Table 6: Simulated interest rates – upward sloping yield curve

Interest Rates	Simulated Mean	Std. Deviation
Short rate	4.2542	1.1589
1-year	4.3093	1.1122
5-year	4.5017	0.7846
10-year	4.6373	0.6759
30-year	4.6954	0.6856

Table 7 shows the results for the four nominal strategies, assuming an upward sloping and more volatile nominal yield curve. The average cost of the debt now rises as the proportion of 30-year nominal bonds in the issuance strategy is increased and strategy 4 – composed of only 30-year nominal bonds – is now the most costly strategy. Note also that the average cost of all four issuance strategies is larger than in the previous example because the average interest rates are now higher. The consequence of changing the yield curve assumptions for the comparison of the issuance strategies is also highlighted in Figure 10, which shows the average interest cost of financing £1 of borrowing requirement for the four issuance strategies. Here, the average interest cost for the issuance strategies increases with the share of 30-year nominal bonds that they contain.

The risk properties of the issuance strategies worsen as the share of 30-year bonds is increased. Both the standard deviation of the debt cost ratio and the debt cost-at-risk measure are largest for the issuance strategy composed of only 30-year bonds. Figure 11 shows that this result is partly attributed to the relatively higher interest rate volatility associated with 30-year nominal bonds, as issuance strategies with a larger

[8] To generate this new nominal yield curve we fix $\lambda = 11.6$.

share of these bonds will tend to exhibit correspondingly higher interest rate volatility and debt cost volatility.

Figure 9

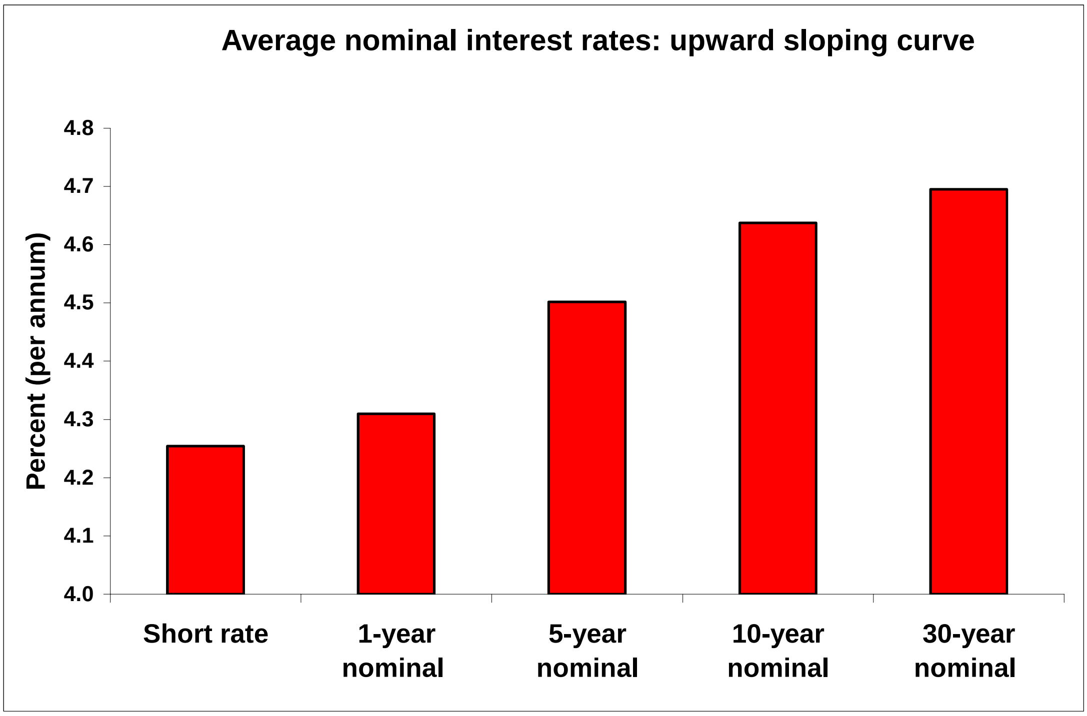


Table 7: Summary of simulation results for nominal issuance strategies assuming an upward sloping yield curve

Strategy	Strategy1	Strategy 2	Strategy 3	Strategy 4
Debt cost/GDP at t=500				
Mean	1.5040	1.5220	1.5212	1.5668
Std deviation	0.1832	0.1798	0.1969	0.2136
95th percentile	1.8232	1.8124	1.8588	1.9264
Debt cost/GDP over the interval t=400 to t= 500				
Mean Mean	1.5256	1.5048	1.5252	1.5635
Mean Std deviation	0.1813	0.1786	0.1963	0.2192
Mean 95th percentile	1.8120	1.8068	1.8531	1.9338

Note: See Table 5 for definition of statistics. Figures are quarterly, annualised and expressed in percentage points

Thus, like any model, the results obtained from the simulation exercises are sensitive to the assumptions made in the modelling process, and in particular the assumptions about the term structure of interest rates, and hence the relative cost of issuing the respective bonds.

Figure 10

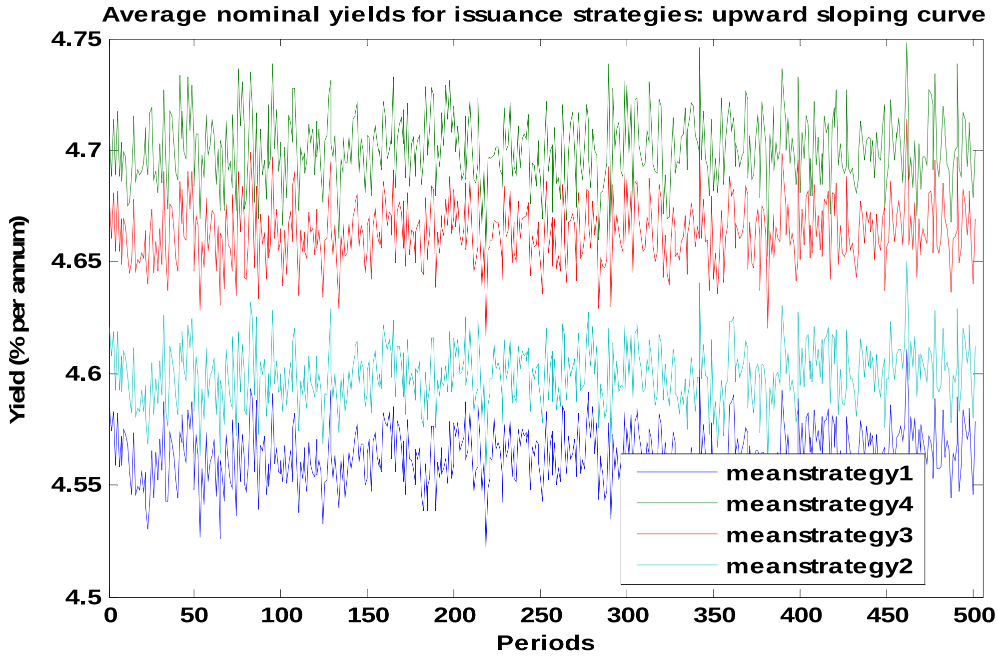
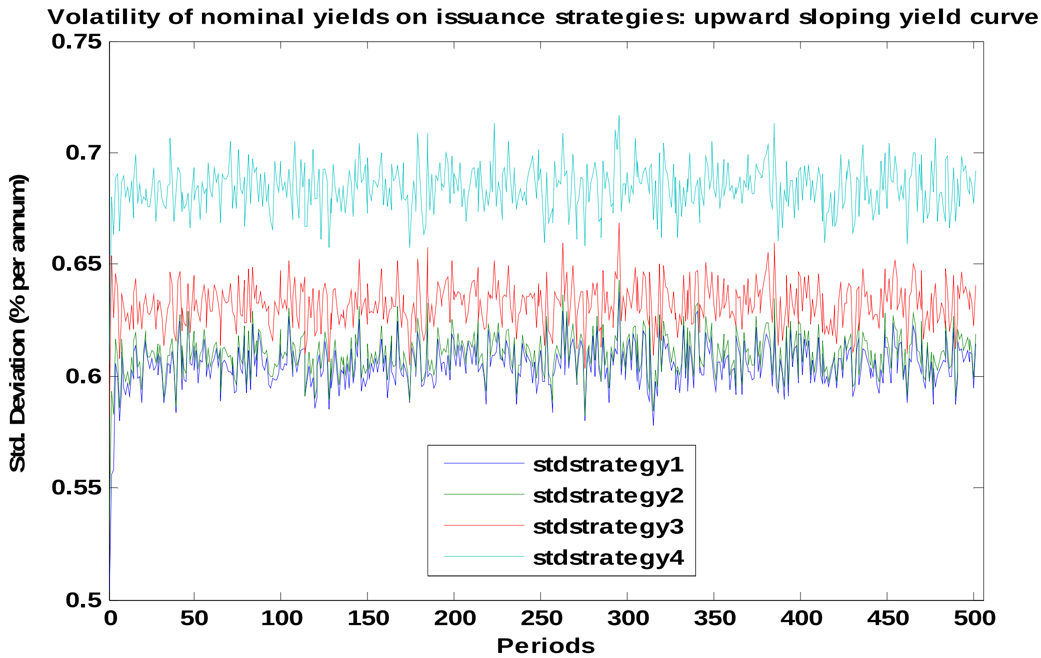


Figure 11



3.2 Results for issuance strategies with inflation-linked bonds

The introduction of inflation-linked bonds raises a few issues for the cost-risk evaluation of the issuance strategies because their cash flow cost comprises two elements: the uplifted coupon payments and the uplift on the principal sum borrowed (see Appendix for an explanation of the cash flow structure of an inflation-linked bond). The inflation compensated coupon payments are paid in the same manner and at the same time as coupon payments on nominal bonds, but the compensation on the principal is paid at redemption. For any given inflation-linked bond the inflation uplift on the principal will be a relatively large sum in comparison to the inflation uplifted interest payments in any given period. This would tend to make the cash flow debt cost of the inflation-linked bonds more volatile than the coupon payments on the nominal bonds.

An alternative way of accounting for the inflation compensation on the principal would be to treat it on an accruals basis so that it is added to the inflation uplifted coupon payments over the life of the bond rather than at the maturity date of the bond. Results with both of these forms of accounting for the uplift on the principal are presented. Table 8 contains the results when the inflation uplift on the principal is paid on the maturity date of the bond and Table 9 presents the results when the inflation uplift on the principal is treated on an accruals basis.

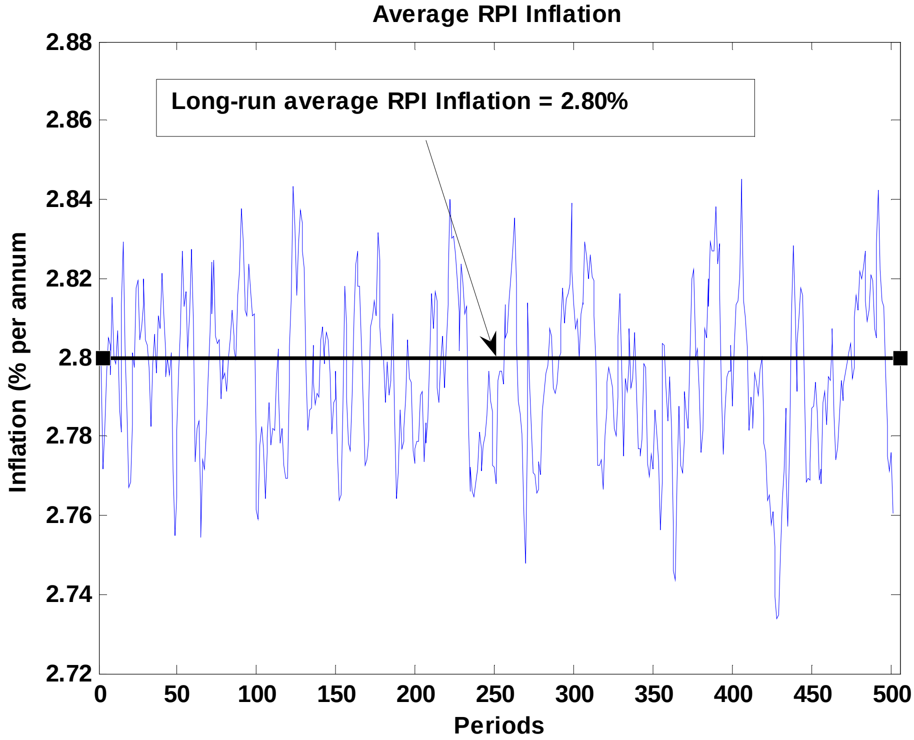
Table 8: Summary of simulation results for issuance strategies with shares of inflation-linked bonds (compensation on principal paid at maturity)

Strategy	Strategy 5	Strategy 6	Strategy 7	Strategy 8
Debt cost/GDP at t=500				
Mean	1.5672	2.0852	1.2996	1.2124
Std deviation	0.3158	0.3267	0.4080	0.8608
95th percentile	2.0984	2.6308	1.9740	2.6500
Debt cost/GDP over the interval t=400 to t= 500				
Mean Mean	1.3579	1.3708	1.3571	1.3047
Mean Std deviation	0.3023	0.2969	0.4000	0.8495
Mean 95th percentile	1.8580	1.8646	2.0297	2.7028

Note: See Table 5 for definition of statistics. Figures are quarterly, annualised and expressed in percentage points.

When the inflation uplift on the principal is paid on the maturity date of the bond it is more meaningful to compare issuance strategies using the average statistics computed over the last 100 periods of the simulation rather than comparing statistics from the last period of the simulation as the latter statistics could be unduly influenced by the uplift on the principal in that period. Overall, the results show that issuing 30-year inflation-linked bonds instead of 30-year nominal bonds reduces the debt cost (as a share of GDP) and as the proportion of 30-year inflation-linked bonds is increased in the issuance strategies the debt cost is further reduced. These results are obtained because, on average, 30-year inflation-linked bonds are relatively less expensive than 30-year conventional bonds. This result is partly a consequence of the tendency for the actual RPI index to grow, on average, at a slightly different (and slower rate) than the expected RPI index which grows at the assumed constant long run average RPI inflation. Figure 12 shows how the simulated average RPI inflation differs from the constant long run average RPI inflation in the model.

Figure 12



In contrast, issuing 30-year inflation linked bonds instead of 30-year nominal bonds makes the debt cost more risky and the risk profile of the debt strategies worsens as the share of 30-year inflation-linked bonds increases in the issuance strategy. Both the standard deviation of the debt cost ratio and the debt cost ratio-at-risk measure are now larger for all the issuance strategies than they were under all of the conventional issuance strategies. The poorer risk characteristics for these strategies are to be expected because the debt cost is influenced directly by the volatility of RPI inflation. Further the risk characteristics are also adversely affected by the typically large cash flow payments at redemption related to the uplift compensation on the principal.

Average cost and risk results similar to those described above are also obtained when the inflation uplift on the principal is treated on an accruals basis. Table 9 shows that the average debt costs of all the issuance strategies are lower than those for the conventional issuance strategies and the average cost is lowest when the share of 30-year inflation linked bonds in the issuance strategy is 100 percent. This result once more follows because, even in the absence of an inflation risk premium on conventional bonds in the model, the behaviour of the actual RPI inflation is, on average, sufficiently different from the assumed constant expected RPI inflation to make 30-year inflation-linked bonds cheaper than 30-year nominal bonds, as was explained above.

The risk characteristics of the issuance strategies are similar to those in the previous example where the inflation uplift on the principal is calculated when the inflation-linked bond matures. Both the standard deviation of the debt cost ratio and the debt cost ratio-at-risk measure are now larger for all the issuance strategies than they were for the conventional issuance strategies, indicating that adding the inflation-linked bonds to the issuance strategies have led to poorer risk characteristics.

However, it is worth noting that the risk measures all have lower values relative to the previous example when the inflation compensation on the principal was paid at redemption. This result follows naturally because the inflation uplift on the principal is now paid over the life of the bond rather than when it matures, thereby smoothing the cash flow payments.

Table 9: Summary of simulation results for issuance strategies with shares of inflation-linked bonds (with accrued compensation on principal)

Strategy	Strategy 5	Strategy 6	Strategy 7	Strategy 8
Debt cost/GDP at t=500				
Mean	1.4200	1.4160	1.4112	1.3752
Std deviation	0.2136	0.2053	0.2120	0.2740
95^{th} percentile	1.8032	1.7704	1.7860	1.8304
Debt cost/GDP over the interval t=400 to t= 500				
Mean Mean	1.4139	1.4189	1.4076	1.3858
Mean Std deviation	0.2113	0.2060	0.2160	0.2754
Mean 95^{th} percentile	1.7810	1.7743	1.7833	1.8538

Note: See Table 5 for definition of statistics. Figures are quarterly, annualised and expressed in percentage points

It is useful to reiterate that an important limitation of the simulation model is that it does not incorporate any feedback from the issuance strategies onto the cost of issuance. This is because it is assumed that interest rates on the respective bonds are not sensitive to the relative supplies of the bonds. In practice this is not generally so as alterations in the relative supply of bonds tend to influence their yields. In a simulation model like this, ignoring the feedback of issuance on interest rates has the unfortunate consequence of leading towards extreme outcomes. When it is assumed that the yield curve is inverted at the long end, as is the case in the main examples presented above, the model suggests that it is cheaper to issue only long conventional bonds (when the focus is only on conventional issuance strategies) and long inflation-linked bonds. Future developments of the simulation model will attempt to correct for this shortcoming by incorporating a suitable feedback mechanism that allows interest rates to respond to the relative supply of bonds.

4 Summary and concluding remarks

This paper describes the stochastic debt strategy simulation model that the DMO is developing. The model is made up of three main components. The first segment is a macroeconomic model comprising five equations for the output gap, the Government’s net primary financing requirement, CPI and RPI inflation and the central bank’s policy rate - the short interest rate. The macroeconomic model is fairly simple in its construction and it is intended to capture in a highly stylised fashion some of the main features of the UK economy over the most recent decade or so and features that are relevant for an analysis of the Government's debt strategies.

Future work will endeavour to make the macroeconomic model more realistic than it currently is. The dynamics of the model are quite simple, with most of the equations having a simple modified auto-regressive structure. Also the purely backward-looking nature of the model can be modified to allow for forward looking terms, in the true spirit of the new-Keynesian models that have been developed and used for the analysis of monetary policy. A further enhancement would be to make the model structure sufficiently flexible to accommodate extreme adverse economic scenarios, such as, for example, a period of deflation or stagflation.

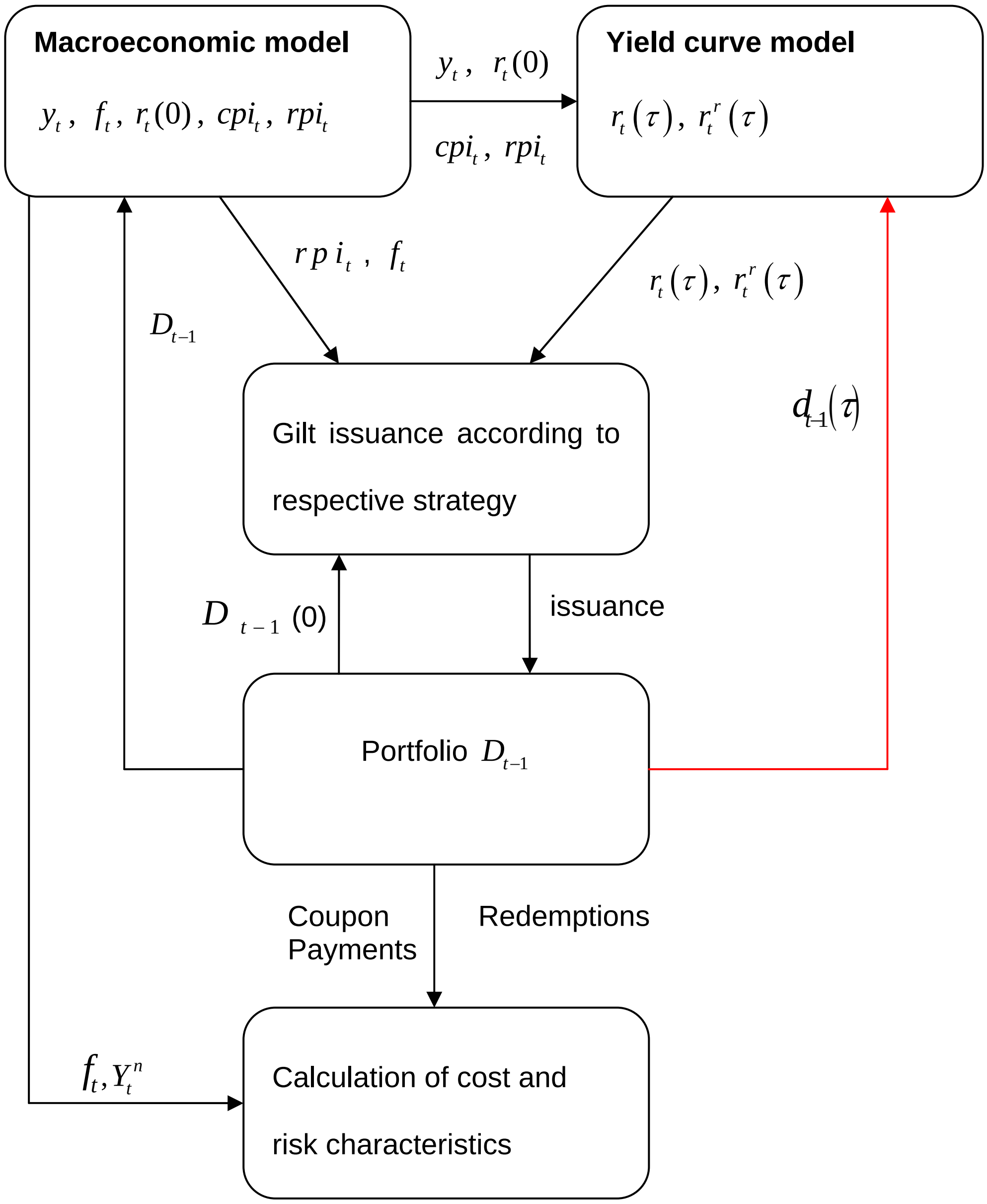
The second component of the simulation model comprises the nominal and real yield curve specifications, which are used to determine the interest rates on nominal bonds and inflation-linked bonds that can be issued by the Government. The Nelson and Siegel functional form is used to model the nominal yield curve and the real yield curve is derived from the nominal yield curve under the assumption of fixed long term inflation expectations. A useful extension of the current specification of the yield curve would be the incorporation of the relative supply of bonds in order to account for the likely influence of changing relative bond supplies on the term structure of interest rates. Figure 13 gives a graphical overview of what the simulation model would look like with this supply-side extension.

The third part of the simulation model is the debt strategy simulation engine through which the Government's borrowing requirement is met by a set of fixed issuance strategies. The debt strategy simulation engine allows these issuance strategies to be compared on the basis of their cost and risk characteristics as it generates the cost distributions associated with each strategy. Assuming that issuance strategies remain fixed over time is clearly a simplification of the debt management process. However examining fixed issuance strategies is a useful starting point in debt strategy simulation modelling because it allows us to understand and illustrate how sensitive simulation results are to variations in some of the important assumptions underpinning the model.

The paper also illustrates how the simulation model can be used to compare issuance strategies. Importantly, the examples discussed have highlighted how the results obtained are sensitive to crucial assumptions made. In particular, assumptions made about the term structure of interest rates determine the relative cost and risk of the respective bonds and therefore the relative cost and risk of the issuance strategies, which are effectively portfolios of the different bonds. The absence of any feedback from issuance strategy to the term structure of interest rates is one limitation of the simulation model, as alluded to previously. The consequence of this limitation is that the relative supply of bonds does not affect their yields and therefore there is a tendency for extreme outcomes – all long nominal or inflation-linked bond strategies -to be preferred.

On its own, this simulation model cannot determine what should be the Government's preferred debt issuance strategy. That can only be determined on the basis of information about the Government's cost-risk trade-off preferences and a consideration of the others factors that the UK authorities examine when choosing a given long-term borrowing strategy. However, this simulation model can be used to illustrate the medium to long-term conditions under which various issuance strategies would lead to desirable outcomes (cheaper and less risky funding) for the Government.

Figure 13: Graphical representation of the simulation model with supply-side extension



Note: y_t: output gap; $r_t(\tau)$ and $r_t^r(\tau)$: nominal and real interest rates respectively with maturity λ; cpi_t and rpi_t: CPI and RPI inflation respectively; f_t: primary net financing requirement; D_{t-1}: debt stock; D_{t-1}(0): maturing debt stock; $d_{t-1}(\tau)$: share of debt of maturity λ in debt stock and Y_t^n: nominal GDP.

In practice, the debt management process entails the consideration of several factors, which affect the long-term cost and risk of managing the Government's debt portfolio. Further, as part of a prudent debt management strategy the debt management authorities take various steps that contribute to the mitigation of various sources of risk that may adversely affect the Government's issuance programme and if not contained could result in higher long-term borrowing costs. [9] Several of these sources of risk are not captured explicitly in the simulation model or are excluded altogether.

One key measure taken is the adoption of an open, transparent and predictable approach to the annual issuance programme. This commitment to transparency and predictability in the issuance programme reflects the Government's judgement that such an approach will reduce the long run borrowing costs of the Government because it lowers the risk premium that investors demand from the issuer as compensation for the unpredictability in issuance supply to the market.

An important assumption that is made when considering the Government's debt strategy is that the Government will be a repeat borrower and therefore its borrowing horizon is indefinite. This implies that the Government will want to ensure that it will be able to raise funds in a sustainable manner into the future. From this perspective, the promotion and maintenance of an efficient and liquid gilts market matters to the Government, as well as having a well-diversified investor base that reflects the prospective demand for gilts under a variety of conditions. The UK debt management authorities adopt measures, such as for example the maintenance of a well-functioning primary dealership arrangement, the issuance of gilts at key maturities along the yield curve and the building up of stocks to benchmark sizes, in order to promote and maintain a liquid and efficient gilts market and also to reach as broad a spectrum of investors as possible. Even when it may not have a need to borrow funds, for example at times of budgetary surpluses as was the case in financial year 2000-01, the Government may still continue its issuance programme, so as to sustain the gilts market infrastructure and prevent liquidity from drying-up altogether in some segments of the gilts market. UK authorities judge that over the long-term these measures together help to lower the Government's financing costs by helping it to capture liquidity and benchmark premia. These important considerations are not captured in the simulation model.

Finally, consideration of the Government's risk preferences is also important when determining the issuance programme. Ceteris paribus, the Government would like to have a prudent debt portfolio structure such that in the event of adverse shocks to the government finances, the debt portfolio should not exacerbate further the strains on the Government's resources, but should help to mitigate some of those strains. In other words, the Government's debt portfolio should be structured so as to possess adequate fiscal-smoothing properties. The implication of taking into account the Government's risk preferences, as well as the other factors previously discussed, when determining its debt strategy is that the Government naturally has a proclivity to choose issuance strategies and a debt portfolio structure which are diversified both in terms of their maturity structure for nominal gilts and their composition in terms of the proportion of the various debt instruments, which in the present environment means the split between nominal and inflation-linked gilts. Such a well-diversified issuance and portfolio structure provide a prudent risk mitigation approach to debt management as, to the extent that different debt instruments have different risk and cost characteristics, they therefore help to insure the Government in the face of a variety of shocks to its finances. Hence, the preferred issuance strategies suggested by the simulation illustrations will need to be modified in practice. At present the Government has a default issuance strategy for nominal gilts in which issuance

[9] **DMO Annual Review 2003-04,** Chapter 7, pp.31-43 provides a detailed exposition of the UK debt management strategy and the various factors that are taken into consideration when determining the debt management strategy and annual financing remit each year.

across the three maturity brackets – short, medium and long[10] – is split approximately equally. In addition, the debt portfolio is further diversified by the regular issuance of inflation-linked bonds, which account for approximately twenty-five percent of the outstanding stock of marketable government debt.

[10] The maturity brackets are defined as follows: short - 1-7 years maturity, medium - 7-15 years maturity and long - over 15 years maturity.

References

Bergström, P., and A. Holmlund (2000), "A simulation model framework for government debt analysis," *mimeograph*, The Swedish National Debt Office.

Bergström, P., A. Holmlund and S. Lindberg (2002), "The SNDO's simulation model for government debt analysis," *mimeograph*, The Swedish National Debt Office.

Bolder, D.J. (2002) "Towards a more complete debt strategy simulation framework," Bank of Canada *Working Paper* 2002-13.

Bolder, D.J. (2003), "A stochastic simulation framework for the government of Canada's debt strategy," Bank of Canada *Working Paper* 2003-10.

Cox, J.C. and J.E. Ingersoll, and S.A. Ross (1985), "A theory of the term structure of interest rates," *Econometrica*, Vol. 53, No. 2, pp. 385-407.

Danmarks Nationalbank (2005) *Danish government borrowing and debt 2004.*

Deacon, M., A. Derry and D. Mirfendereski (2004), *Inflation-indexed securities*, John Wiley & Sons, Ltd, England.

Diebold, F.X., M. Piazzesi, and G. D. Rudebusch (2005), "Modelling bond yields in Finance and Macroeconomics," *American Economic Review, Papers and Proceedings*, Vol. 95, Issue 2, pp. 415 – 420.

Diebold, F.X., and C. Li (2006), "Forecasting the term structure of government bond yields," *Journal of Econometrics*, Vol. 130, pp 337-364.

Diebold, F.X., and G. D. Rudebusch and S.B. Aruoba (2006), "The macroeconomy and the yield curve: a dynamic latent factor approach," *Journal of Econometrics*, Vol. 131, pp 309-338.

Goodwin, T.H. (1993), "Business-cycle analysis with a Markov-switching model," *Journal of Business and Economic Statistics*, Vol. 11, # 3, pp 331 -339.

Hamilton, J.D. (1989), "A new approach to the economic analysis of non-stationary time series and the business cycle," *Econometrica*, Vol. 57, # 2, pp. 357-384.

Harvey, A.C. (1989), *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge University Press, Cambridge.

HM Treasury (2006) *Debt and Reserves Management Report 2006-07.*

HM Treasury (2005) *Pre-Budget Report 2005.*

Litterman, R. and J. Scheinkman (1991), "Common factors affecting bond returns," *Journal of Fixed Income*, pp. 54-61.

Nelson, C.R., and A.F. Siegel (1987), "Parsimonious modelling of yield curves," *Journal of Business*, Vol. 60, # 4.

Risbjerg, L. and A. Holmlund (2005), "Analytical Framework for Debt and Risk Management," in *Advances in Risk Management of Government Debt*, OECD, Paris, France.

The United Kingdom Debt Management Office, *DMO Annual Review 2003-2004*.

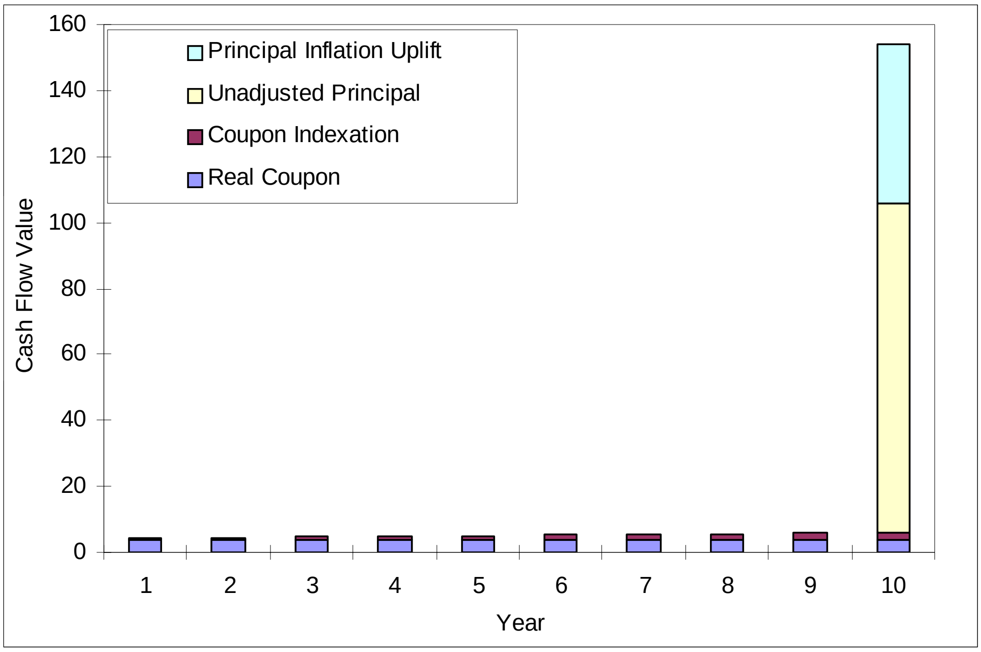
Appendix: Cash flow structure of inflation-linked bonds

The design of inflation-linked bonds, in practice, can have various forms. Inflation-linked bonds issued by the UK Government take the form of a *capital indexed bond (CIB)*. CIBs are, by far, the most popular design of inflation-linked bonds issued by governments. A CIB has a fixed real coupon rate and the nominal principal rises with inflation. The coupon payment in a given period is calculated as the product of the real coupon rate and the inflation-compensated principal. The inflation-compensated principal is paid on the maturity date of the bond when it is redeemed. The cash flow structure of a CIB is illustrated in Figure A1 and Table A1, which are both reproduced, with the kind permission of Mark Deacon, from the book he has co-authored with Andrew Derry and Dariush Mirfendereski[1].

Table A1: An example of Capital Index Bond's cash flows

Year	Real Coupon (1)	Inflation (2)	Compounded Inflation (3)	Coupon Indexation (4) = (5) - (1)	Coupon Payment (5) = (1) x (3)	Redemption Payment (6) = 100 x (3)
1	4.00	6.00	1.0600	0.24	4.24	
2	4.00	5.50	1.1183	0.47	4.47	
3	4.00	5.00	1.1742	0.70	4.70	
4	4.00	5.00	1.2329	0.93	4.93	
5	4.00	4.00	1.2822	1.13	5.13	
6	4.00	3.50	1.3271	1.31	5.31	
7	4.00	3.00	1.3669	1.47	5.47	
8	4.00	3.00	1.4079	1.63	5.63	
9	4.00	2.50	1.4431	1.77	5.77	
10	4.00	2.50	1.4792	1.92	5.92	147.92

Figure A1: Capital Index Bond's cash flows



[1] See Deacon, Derry and Mirfendereski (2004)