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PWLB lending facility

Formula for calculating premium / discounts on fixed rate loans

The formulae set out in this document relate to half-yearly loans – formulae for annual loans are available on request.

The following elements are used in the formulae –

id = discount rate / 200
il = loan rate / 200
D = days since previous interest payment date
N = number of half years (or parts of half years) in unexpired period
A1 = Annuity factor for unexpired period at discount rate for unexpired period
A2 = Annuity factor for unexpired period at loan rate for unexpired period
PV1 = Present Value of 1 at discount rate for unexpired period = $\frac{1}{A1}$

Discount rate - this rate is taken from the 'premature repayment' set of rates in force when the repayment is agreed for a loan repayable by the same method over the same remaining term as the loan being repaid.
Loan rate – this is rate payable on the loan being repaid

The formulae for each repayment method are as follows –

Maturity Loans

$$(1+\frac{id(D)}{182.5})(\frac{il}{A1}+\text{PV1})-(1+\frac{il(D)}{182.5})$$

Equal Instalment of Principal (EIP) Loans

$$(1+\frac{id(D)}{182.5})(\frac{1}{N(A1)}(1-\frac{il}{id})+(\frac{il}{id}))-(1+\frac{il(D)}{182.5})$$

Annuity Loans

$$(1+\frac{id(D)}{182.5})(\frac{A2}{A1})-(1+\frac{il(D)}{182.5})$$

Note – if the result of the calculation is negative a discount is allowed and if the result is positive a premium is payable. If the discount rate and loan rate are identical then the premium / discount is zero.

Worked example 1 – Maturity Loan

Loan to be repaid on **2 October 2015**

Discount rate – 2.24% (rate determined at 09:30 on 2 October 2015)
Loan rate – 3.14%
Date of Final Payment – 12 March 2063 (unexpired period – 47 ½ years)
Amount to be repaid - £5,000,000.00

$id = 2.24 / 200 = \mathbf{0.0112}$
$il = 3.14 / 200 = \mathbf{0.0157}$
D = **20** (12 September to 2 October 2015)
N = **95** half years

$$A1 = \frac{id}{1-(1+id)^{-n}} = \frac{0.0112}{1-(1+0.0112)^{-95}} = \frac{0.0112}{1-0.347119659} = \frac{0.0112}{0.652880341}$$
$$= \mathbf{0.0171547515}$$

$$PV1 = (1+id)^{-n} = (1+0.0112)^{-95} = \mathbf{0.347119659}$$

Premium / Discount

$$= \left(1+\frac{id(D)}{182.5}\right)\left(\frac{il}{A1} + PV1\right) - \left(1+\frac{il(D)}{182.5}\right)$$

$$= \left(1+\frac{0.0112(20)}{182.5}\right)\left(\frac{0.0157}{0.0171547515} + 0.347119659\right) - \left(1+\frac{0.0157(20)}{182.5}\right)$$

$$= \left(1+\frac{0.224}{182.5}\right)\left(0.9151983344 + 0.347119659\right) - \left(1+\frac{0.314}{182.5}\right)$$

$$= (1.00122739726)(1.2623179934) - (1.001720547945)$$

$$= 1.2638673590 - 1.0017205480 = \mathbf{0.262146811}$$

The result is positive, therefore a **Premium** is payable

= £5,000,000.00 * 0.262146811= 1,310,734.055 = **£1,310,734.06**

Worked example 2 – EIP Loan

Loan to be repaid on **2 October 2015**

Discount rate – 1.66% (rate determined at 09:30 on 2 October 2015)
Loan rate – 2.66%
Date of Final Payment – 14 September 2035 (unexpired period – 20 years)
Amount to be repaid - £5,000,000.00

$id = 1.66 / 200 = \mathbf{0.0083}$
$il = 2.66 / 200 = \mathbf{0.0133}$
D = **18** (14 September to 2 October 2015)
N = **40** half years

$$A1 = \frac{id}{1-(1+id)^{-n}} = \frac{0.0083}{1-(1+0.0083)^{-40}} = \frac{0.0083}{1-0.718471115} = \frac{0.0083}{0.281528885}$$
$$= \mathbf{0.0294818771}$$

Premium / Discount

$$= (1 + \frac{id(D)}{182.5})(\frac{1}{N(A1)}(1 - \frac{il}{id}) + (\frac{il}{id})) - (1 + \frac{il(D)}{182.5})$$

$$= (1 + \frac{0.0083(18)}{182.5})(\frac{1}{40(0.0294818771)}(1 - \frac{0.0133}{0.0083}) + (\frac{0.0133}{0.0083})) - (1 + \frac{0.0133(18)}{182.5})$$

$$= (1 + \frac{0.1494}{182.5})(\frac{1}{1.179275084}(1 - \frac{0.0133}{0.0083}) + (\frac{0.0133}{0.0083})) - (1 + \frac{0.2394}{182.5})$$

$$= (1 + \frac{0.1494}{182.5})(0.8479785705(1 - 1.6024096386) + (1.6024096386)) - (1 + \frac{0.2394}{182.5})$$

$$= (1.0008186301)((0.8479785705(-0.6024096386) + (1.6024096386)) - (1.0013117808)$$

$$= (1.0008186301)(-0.5108304642 + 1.6024096386) - (1.0013117808)$$

$$= (1.0008186301)(1.0915791744) - (1.0013117808)$$

$$= 1.0924727740 - 1.0013117808 = 0.0911609932$$

The result is positive, therefore a **Premium** is payable

= £5,000,000.00 * 0. 0911609932= 455,804.966 = **£455,804.97**

Worked example 3 – Annuity Loan

Loan to be repaid on **2 October 2015**

Discount rate – 2.12% (rate determined at 09:30 on 2 October 2015)
Loan rate – 3.27%
Date of Final Payment – 10 September 2045 (unexpired period – 30 years)
Amount to be repaid - £5,000,000.00

$id = 2.12 / 200 = \mathbf{0.0106}$
$il = 3.27 / 200 = \mathbf{0.01635}$
D = **22** (10 September to 2 October 2015)
N = **60** half years

$$A1 = \frac{id}{1-(1+id)^{-n}} = \frac{0.0106}{1-(1+0.0106)^{-60}} = \frac{0.0106}{1-0.5311807966} = \frac{0.0106}{0.4688192034}$$
$$= \mathbf{0.0226099953}$$

$$A2 = \frac{il}{1-(1+il)^{-n}} = \frac{0.01635}{1-(1+0.01635)^{-60}} = \frac{0.01635}{1-0.3779222919} = \frac{0.01635}{0.6220777081}$$
$$= \mathbf{0.0262828900}$$

Premium / Discount

$$= \left(1 + \frac{id(D)}{182.5}\right)\left(\frac{A2}{A1}\right) - \left(1 + \frac{il(D)}{182.5}\right)$$

$$= \left(1 + \frac{0.0106(22)}{182.5}\right)\left(\frac{0.0262828900}{0.0226099953}\right) - \left(1 + \frac{0.01635(22)}{182.5}\right)$$

$$= \left(1 + \frac{0.2332}{182.5}\right)\left(1.1624456198\right) - \left(1 + \frac{0.3597}{182.5}\right)$$

$$= \left(1 + 0.0012778082\right)\left(1.1624456198\right) - \left(1 + 0.0019709589\right)$$

$$= \left(1.0012778082\right)\left(1.1624456198\right) - \left(1.0019709589\right)$$

$$= \left(1.1639310023\right) - \left(1.0019709589\right) = 0.1619600434$$

The result is positive, therefore a **Premium** is payable

= £5,000,000.00 * 0. 0.1619600434= 809,800.217 = **£809,800.22**